केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN

आज़ादी का अमृत महोत्सव

शिक्षा एवं प्रशिक्षण का आंचलिक संस्थान, चंडीगढ़
ZONAL INSTITUTE OF EDUCATION AND TRAINING, CHANDIGARH

अध्ययन सामग्री / STUDY MATERIAL

शैक्षिक सत्र / Session – 2022-23

कक्षा / Class – बारहवीं/ TWELVE(XII)

विषय / Subject - अनुप्रयुक्त गणित/ APPLIED MATHEMATICS

विषय कोड / Subject Code - 241

तैयारकर्ता / Prepared By
पी.जी.टी (गणित)
चंडीगढ़, देहरादून,दिल्ली , गुरुग्राम एवं जम्मू संभाग
अनुप्रयुक्त गणित के लिए अध्ययन सामग्री और मॉड्यूल
के विकास पर 3 दिवसीय कार्यशाला के दौरान
01 नवम्बर से 03 नवम्बर 2022 तक

PGT(MATHEMATICS)
CHANDIGARH, DEHRADUN, DELHI, GURUGRAM & JAMMU REGION
DURING 3 DAY WORKSHOP ON DEVELOPMENT OF STUDY MATERIAL
& MODULES FOR APPLIED MATHEMATICS
From 01 November to 03 November 2022

शिक्षा एवं प्रशिक्षण का आंचलिक संस्थान, चंडीगढ़
ZONAL INSTITUTE OF EDUCATION AND TRAINING, CHANDIGARH
सेक्टर-33 सी, चंडीगढ़ / **SECTOR-33C, CHANDIGARH**

वेबसाइट / Website : zietchandigarh.kvs.gov.in

ई-मेल / e-mail :kvszietchd@gmail.com दूरभाष / Phone : 0172-2621302, 2621364

हमारे संरक्षक

श्रीमती निधि पांडे, आईआईएस आयुक्त	**Mrs. NIDHI PANDEY, IIS** **COMMISSIONER**
श्री एन. आर. मुरली संयुक्त आयुक्त (शैक्षिक)	**Mr. N R MURALI** **JOINT COMMISSIONER(ACADEMICS)**
श्री सत्य नारायण गुलिया संयुक्त आयुक्त (वित्त)	**Mr. SATYA NARAIN GULIA** **JOINT COMMISSIONER (FINANCE)**
श्रीमती अजीता लोंग्जम संयुक्त आयुक्त (प्रशासन-I)	**Mrs. AJEETA LONGJAM** **JOINT COMMISSIONER (ADMIN-I)**
डॉ. जयदीप दास संयुक्त आयुक्त (प्रशासन)	**Dr. JAIDEEP DAS** **JOINT COMMISSIONER (ADMIN)**
डा. पी देवाकुमार संयुक्त आयुक्त (कार्मिक)	**Dr. P. DEVAKUMAR** **JOINT COMMISSIONER (PERS)**
सुश्री चंदना मंडल संयुक्त आयुक्त (प्रशिक्षण)	**Ms. CHANDANA MANDAL** **JOINT COMMISSIONER (TRAINING)**

निदेशक महोदय का संदेश

विद्यार्थियों की शैक्षिक प्रगति को ध्यान में रखते हुए उपयोगी अध्ययन सामग्री उपलब्ध कराना हमारा महत्त्वपूर्ण उद्देश्य है। इससे न केवल उन्हें अपने लक्ष्य को प्राप्त करने में सरलता एवं सुविधा होगी बल्कि वे अपने आंतरिक गुणों एवं अभिरुचियों को पहचानने में सक्षम होंगे। बोर्ड परीक्षा में अधिकतम अंक प्राप्त करना हर एक विद्यार्थी का सपना होता है। इस संबंध में तीन प्रमुख आधार स्तंभों को एक कड़ी के रूप में देखा जाना चाहिए- अवधारणात्मक स्पष्टता, प्रासंगिक परिचितता एवं आनुप्रयोगिक विशेषज्ञता।

राष्ट्रीय शिक्षा नीति 2020 के उद्देश्यों की मूलभूत बातों को गौर करने पर यह तथ्य स्पष्ट है कि विद्यार्थियों की सोच को सकारात्मक दिशा देने के लिए उन्हें तकनीकी आधारित समेकित शिक्षा के समान अवसर उपलब्ध कराए जाएं। बोर्ड की परीक्षाओं के तनाव और दबाव को कम करने के उद्देश्य को प्रमुखता देना अति आवश्यक है।

यह सर्वमान्य है कि छात्र-छात्राओं का भविष्य उनके द्वारा वर्तमान कक्षा में किए गए प्रदर्शन पर ही निर्भर करता है। इस तथ्य को समझते हुए यह अध्ययन सामग्री तैयार की गई है। उम्मीद है कि प्रस्तुत अध्ययन सामग्री के माध्यम से वे अपनी विषय संबंधी जानकारी को समृद्ध करने में अवश्य सफल होंगे।

शुभकामनाओं सहित।

मुकेश कुमार
उपायुक्त एवं निदेशक

COURSE DIRECTOR

Shri MUKESH KUMAR
DEPUTY COMMISIIONER & DIRECTOR, ZIET, CHANDIGARH

COURSE COORDINATOR

Shri RAJIV RANJAN
TRAINING ASSOCIATE(Maths), ZIET, CHANDIGARH

RESOURCE PERSON

Shri ATUL KUMAR YADAV
PGT(Maths) , K V SECTOR 31, CHANDIGARH

PARTICIPANTS

S.No.	Name of PGT(Maths)	Name of KV	Region
1	Mrs NAVNEET KAUR	Sec. 29 Chandigarh	Chandigarh
2	Mr SURAJ BHAN SINGH RANA	3 BRD Chandigarh	Chandigarh
3	Mr. AMIT CHANDAN	Pathankot No. 3	Chandigarh
4	Mr ROHIT KUMAR	Ferozpur No. 2	Chandigarh
5	Ms. MONU SEHGAL	Jalalabad	Chandigarh
6	Mr. YOG RAJ SHARDA	Gurdaspur	Chandigarh
7	Mr. SHIV KUMAR	Nangal	Chandigarh
8	Sh. MANISH SEMWAL	Uttarkashi	Dehradun
9	Mr CHANDRA PRAKASH JOSHI	ALMORA	Dehradun
10	Smt KUSUM LATA NEGI	Bageshwar	Dehradun
11	Mr VIMAL KISHORE PANDEY	Bhimtal	Dehradun
12	Shri RAJESH KUMAR ARYA	HALDWANI(S-2)	Dehradun
13	Mr PERVASH KUMAR	HARDWADAR BHEL	Dehradun
14	Mrs RENU	MUSSORIE	Dehradun
15	Dr. ITI GUPTA	Andrewsganj (1st Shift)	DELHI
16	Shri VIJAYANT GAUR	Delhi Cantt. No.2(1st Shift)	DELHI
17	Mr R D SHARMA	Rohini Sec - 8 (2nd Shift)	DELHI
18	Mr. RAVINDER SINGH	Shalimar Bagh (1st Shift)	DELHI
19	Mr. SATISH KUMAR	Shalimar Bagh (2nd Shift)	DELHI
20	Mrs NIRPESH KUMARI	Ambala No.2	Gurugram
21	Mr. O P SWAMI	Gurgaon AFS I Shift	Gurugram
22	Mr. BHARAT BHUSHAN	Sirsa AFS No.1	Gurugram
23	Smt CHARU SHARMA	Rohtak	Gurugram
24	Mr A P SINGH	Ambala No.3	Gurugram
25	Mrs EKTA	Hisar Cantt	Gurugram
26	Smt KAVITA YADAV	Rewari	Gurugram
27	Mr AMARJIT SINGH	Palampur	Gurugram
28	Mr VEERJI KOUL	NO.I Udhampur	Jammu

GROUP INFORMATION AND WORK ALLOCATION

GROUP 1

S. No.	Name of Members	Designation
1	Mr VEERJI KOUL	PGT(MATHS)
2	Mr AMARJIT SINGH	PGT(MATHS)
3	Mr. AMIT CHANDAN	PGT(MATHS)
4	Sh. MANISH SEMWAL	PGT(MATHS)
5	Mr SURAJ BHAN SINGH RANA	PGT(MATHS)
6	Mr. YOG RAJ SHARDA	PGT(MATHS)
7	Mr ROHIT KUMAR	PGT(MATHS)

GROUP 2

S. No.	Name of Members	Designation
1	Mr R D SHARMA	PGT(MATHS)
2	Mr A P SINGH	PGT(MATHS)
3	Mr RAVINDER SINGH	PGT(MATHS)
4	Mr SATISH KUMAR	PGT(MATHS)
5	Mr KAVITA YADAV	PGT(MATHS)
6	Ms EKTA	PGT(MATHS)
7	Ms CHARU SHARMA	PGT(MATHS)

GROUP-3

S. No.	Name of Members	Designation
1	Mr CHANDRA PRAKASH JOSHI	PGT(MATHS)
2	Shri RAJESH KUMAR ARYA	PGT(MATHS)
3	Mr VIMAL KISHORE PANDEY	PGT(MATHS)
4	Mr. SHIV KUMAR	PGT(MATHS)
5	Mr PARVESH KUMAR	PGT(MATHS)
6	Ms NAVNEET KAUR	PGT(MATHS)
7	Ms NIRPESH KUMARI	PGT(MATHS)

GROUP-4

S. No.	Name of Members	Designation
1	Dr ITI GUPTA	PGT(MATHS)
2	Ms. MONU SEHGAL	PGT(MATHS)
3	Smt KUSUM LATA NEGI	PGT(MATHS)
4	Ms RENU	PGT(MATHS)
5	Mr VIJYANT GUPTA	PGT(MATHS)
6	Mr BHARAT BHUSHAN	PGT(MATHS)
7	Mr O P SWAMI	PGT(MATHS)

Numbers, Quantification and Numerical Applications	Group-1
Matrices &Determinants	Group-1
Differentiation and its Applications	Group-2
Integration and its Applications	Group-3
Differential Equations and Modeling	Group-4
Probability Distributions	Group-3
Inferential Statistics	Group-2
Index Numbers and Time Based Data	Group-3
Financial Mathematics	Group-4
Linear Programming	Group-4

INDEX

APPLIED MATHEMATICS (241)
CLASS XII 2022-23

UNIT-1: NUMBERS, QUANTIFICATION AND NUMERICAL APPLICATIONS

Modulo Arithmetic, Congruence Modulo. Alligation and Mixture, Boats and Streams (upstream and downstream), Pipes and Cisterns, Races and Games. Numerical Inequalities.

UNIT-2: ALGEBRA

Matrices and types of matrices, Equality of matrices, Transpose of a matrix, Symmetric and Skew symmetric matrix, Algebra of Matrices, Determinants, Inverse of a matrix, Solving system of simultaneous equations using matrix method, Cramer's rule .

UNIT- 3 : CALCULUS

DIFFERENTIATION AND ITS APPLICATIONS

Higher Order Derivatives, Application of Derivatives, Marginal Cost and Marginal Revenue using derivatives, Increasing /Decreasing Functions, Maxima and Minima.

INTEGRATION AND ITS APPLICATIONS

Integration, Indefinite Integrals as family of curves, Definite Integrals as area under the curve, Application of Integration.

DIFFERENTIAL EQUATIONS AND MODELING

Differential Equations , Formulating and Solving Differential Equations, Application of Differential Equations.

UNIT- 4 : PROBABILITY DISTRIBUTIONS

Probability Distribution, Mathematical Expectation, Variance, Binomial Distribution, Poison Distribution, Normal Distribution.

UNIT – 5 : INFERENTIAL STATISTICS

Population and Sample, Parameter and Statistics and Statistical Interferences, t-Test (one sample t-test and two independent groups t-test).

UNIT – 6 : INDEX NUMBERS AND TIME BASED DATA

Time Series, Components of Time Series, Time Series analysis for univariate data, Secular Trend, Methods of Measuring trend.

UNIT – 7 : FINANCIAL MATHEMATICS

Perpetuity, Sinking Funds, Calculation of EMI, Calculation of Returns, Nominal Rate of Return, Compound Annual Growth Rate, Linear method of Depreciation.

UNIT – 8 : LINEAR PROGRAMMING

Introduction and related terminology, Mathematical formulation of Linear Programming Problem, Different types of Linear Programming Problems, Graphical method of solution for problems in two variables, Feasible and Infeasible Regions, Feasible and infeasible solutions, optimal feasible solution.

Grade XII (2022-23)

Number of Paper : 1 **Total number of Periods : 240 (35 Minutes Each)**

Time : 3 Hours **Max Marks : 80**

No.	Units	**No. of Periods**	**Marks**
I	Numbers, Quantification and Numerical Applications	30	11
II	Algebra	20	10
III	Calculus	50	15
IV	Probability Distributions	35	10
V	Inferential Statistics	10	05
VI	Index Numbers and Time-based data	30	06
VII	Financial Mathematics	50	15
VIII	Linear Programming	15	08
Total		240	80
Internal Assessment			20

NUMBERS, QUANTIFICATION AND NUMERICAL APPLICATIONS

SOME IMPORTANT RESULTS/CONCEPTS

Modulo Arithmetic:

Eular's Division Lemma:

$$For\ integers\ a, b(\neq 0), we\ have\ a = bq + r, where\ q, r \in \mathrm{Z}\ and\ 0 \leq r < |b|$$

Modulo Arithmetic is the arithmetic of remainders.

$$a\ mod\ b = r$$

- $If\ a = b\ or\ b\ divides\ a\ then\ \boldsymbol{a\ mod\ b = 0}$
- $If\ a < b, then\ \boldsymbol{a\ mod\ b = a}$

Congruence Modulo:

Let R be a relation on the set of integers Z defined as

$$R = \{(a, b): a = b(mod\ n) \forall, a, b \in \mathrm{Z}, n \in \mathrm{N}, n > 1\}$$

SHORT ANSWER TYPE QUESTIONS

1. Write the value of 21 mod 8?
2. Write the value of $3 \oplus_8 12$?
3. Find 11 $\ominus_8$ 3?
4. If 24 = 8(mod x), then what are the possible values of x?
5. Find the equivalence class of 2(mod3).
6. Find $(576 + 789)$mod 9.
7. If 57 = x (mod5), Find least positive value of x.
8. If it is 6:00pm currently. What time (in A.M or P.M) will be in next 1500 hours.
9. Find $-6(mod 7)$.
10. Find subtraction modulo 8 if a and b are 15 and 4 respectively.

ANSWERS

1. 5
2. 7
3. 0
4. 1, 2, 4, 8, 16
5. $\{\ldots, -4, -1, 2, 5, 8, 11, \ldots\}$
6. 6
7. 2
8. 06:00 am
9. 1
10. 3

LONG ANSWER TYPE QUESTIONS

1. Find $6^{12}(mod 7)$.
2. Find the last digit 17^{17}.
3. Find the last two digits of 2^{20}.
4. Using modulo Find the remainder when 672+541 + 383+295 +101+86 is divided by 3
5. Find all the positive integers less than 30 forming the equivalence class of 5 for modulo 7
6. There are 81 boxes with 21 articles in each. When we rearrange all of the articles so that each box has 5 articles, how many articles will be left out without a box?

ANSWERS

1. 1
2. 7
3. 76
4. 2
5. {5, 12, 19, 26}
6. 1

Allegation and Mixture:

$$\boldsymbol{Ratio(R) = \frac{C.P. of\ Dearer\ (d) - Mean\ Price(m)}{Mean\ Price(m) - C.P. of\ Cheaper(c)} = \frac{d - m}{m - c}}$$

- Quantity of Pure Liquid is $\boldsymbol{x\left(1 - \frac{y}{x}\right)^n}$

$$where\ x - Original\ amount, y - taken\ out\ and\ n - Number\ of\ times$$

SHORT ANSWER TYPE QUESTIONS

1. In what ratio must rice at 70/kg be mixed with rice 100/kg so that the mixture is worth 80/kg?
2. A certain quantity of water is mixed with milk priced at Rs 50 per litre. The price of mixture is Rs 40 per litre. Find out the ratio of water and milk in the new mixture.
3. In what ratio should a kind of sugar at 8 Rs/kg be mixed with another kind of sugar at 10 Rs/kg so that the mixture be worth 9 Rs/kg?
4. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is
5. In what ratio must water be mixed with milk costing Rs. 12 per litre to obtain a mixture worth of Rs. 8 per litre?
6. A container contains 40 litres of water. From this container 4 litres of water was taken out and replaced by milk. This process was repeated further two times. How much water is now contained by the container?
7. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?
8. **A mixture of certain quantity of milk with 8 L of water is worth 45 Rs per litre. If pure milk be worth 54 Rs per litre, how much milk is three in the mixture?**
9. **600 g of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?**
10. **In what ratio must a grocer mix rice worth Rs. 30 per kg, so that by selling the mixture at Rs. 34.10 per kg, he may gain 10%?**

ANSWERS

1. 2: 1
2. 1: 4
3. 1:1
4. Rs 18
5. 1:2
6. 29.16 litres
7. 1/5
8. 40 litre
9. 120 gram
10. 3:2

LONG ANSWER TYPE QUESTIONS

1. **A container has 60 L of milk, from his container, 4 L of milk is taken out and replaced with water. If this process is repeated 3 times, then quantity of milk in container left.**
2. In what ratio, water must be added to dilute honey costing 240 per litre so that the resulted syrup would be worth 200?
3. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:
4. Two vessels A and B contain milk and water in the ratio of 5:2 and 2:3. When these mixtures are mixed to form a new mixture containing half milk and half water, they must be taken in the ratio :
5. **Copper and Zinc are in the ratio 2 : 3 in 200 gms of an alloy. The quantity (in grams) of copper to be added to it to make the ratio 3 : 2 is:**
6. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be:
7. **How many litres of water should be added to a 35-litre mixture of HCl and water containing HCl and water in the ratio of 4 : 3 such that the resultant mixture has 60% water in it?**
8. **A shopkeeper deals in milk and 45 litre mixture is to be distributed in Milk & Water in the ratio of 4:1. If 4 litre milk & 3 litre water will be added in the mixture then what will be the new ratio of water and milk?**

ANSWERS

1. 48.78 L
2. 1:5
3. 20%
4. 7 :15
5. 100 gram
6. Rs. 175.50
7. 15 litres
8. 3:10

Boats and Streams:

$$\text{Let the speed of the boat in the still water be } x \text{ km/h and sped of the stream be } y \text{ km/h.}$$

$$\text{Downstream Speed } (\boldsymbol{u}) = \boldsymbol{x} + \boldsymbol{y} \text{ km/h and Upstream Speed} (\boldsymbol{v}) = \boldsymbol{x} - \boldsymbol{y} \text{ km/h}$$

$$\text{Speed of Boat} = \frac{u+v}{2} \text{ and Speed of Stream} = \frac{u-v}{2}$$

SHORT ANSWER TYPE QUESTIONS

1. Ratio between speed of boat in still water to speed of stream is 5 : 2. If 224 km is travelled by downstream in 4 hours then find the difference between speed of boat in still water and speed of stream?
2. Sita can row upstream at 30 km/hr and downstream at 34 km/hr. Find the average of Sita's rate in still water and rate of the current.
3. A boat goes 360 km upstream and returns to the same point in 35 hours. If the speed of current is 3km/h, how much distance boat will cover in still water in 6 hours?
4. A boat can travel 9.6 km upstream in 36 min. If speed of the water current is 20% of the speed of the boat in upstream, then how much time will the boat take to travel 24.64 km downstream?
5. A boat can travel from point A to point B and return back to point A in 9 hours. Speed of the boat in still water is 8 km/h and the speed of the stream is 4 km/h. Find the distance between A and B.
6. The ratio of speed of A and B in still water is 5 : 7. If A and B both start from the same point simultaneously and travel downstream, the distance between them 4 hours later will be 24 km. What will be the distance between them after 6 hours, when they travel in opposite directions simultaneously from the same point in still water?

ANSWERS

1. 24km/h
2. 17 km
3. 126
4. 1.1 hours
5. 27m
6. 216

LONG ANSWER TYPE QUESTIONS

1. The ratio of speed of A and B in still water is 3 : 2. A and B start from the same point in the river, A goes upstream and B goes downstream. After 3 hours the stream stops flowing and A starts rowing in the opposite direction to meet B. How much time after the stream stops flowing does A meet B?
2. A steamer can go 12 km in still water in 25 minutes. One day, it went 11.25 km upstream and returned the same distance in downstream. If the difference between the time taken to travel upstream and downstream was 12.5 minutes, then what was the speed of stream in km per hour?
3. A boat goes a certain distance upstream and comes back downstream to the starting point in 144 min. If the speed of the boat in still water becomes 66.67% of the original, time taken for the same journey will be 224 min. What is the ratio of the speed of boat in still water and speed of current?
4. A man rows to a place 46 km distance and back in 11 hours 30 minutes. He found that he can row 5 km with the stream in the same time as he can row 4 km against the stream. Find the rate of the stream.
5. A man can row 30 km downstream in 3 hours 45 minutes and 11 km upstream in 2 hours 12 minutes. What is the difference between the speed of the man in still water and the speed of the stream?
6. There are 3 points P, Q and R in a straight line, such that point Q is equidistant from points P and R. A man can swim from point P to R downstream in 24 hours and from Q to P upstream in 16 hours. Find the ratio of speed of man in still water to speed of stream?

ANSWERS

1. 15 hours
2. 7.2 km per hour
3. 6:1
4. 0.9 km/h
5. 5
6. 70 km/h

Pipes and Cisterns:

A pipe connected to a tank or cistern which fills it is known as inlet pipe and the pipe connected to the tank which drains or empties it is known as outlet pipe. When a tank is connected to many pipes (inlets and outlets), then the difference between the sum of the work done by inlets and the sum of the work done by outlets gives the filled part of the tank.

- Let a pipe fill a tank in x number of hours, then it can fill (1/x) th portion of the tank in one hour.
- If a pipe can empty a tank in y number of hours,
- then it can empty out (1/y)th portion of the tank in one hour.
- The portion of tank they can fill together in one hour = $(\frac{1}{x} - \frac{1}{y})$th
- If a pipe fills $\frac{1}{x}$ part of a tank in 1 hour, then the time taken by the pipe to fill the tank completely is x hours
- Two pipes can fill a tank in x and y hours respectively. If both the pipes are opened simultaneously, then time taken by both the pipes to fill the tank is $\frac{xy}{x+y}$ hours.
- If two pipes A and B together can fill a tank in x hours and the pipe A alone can fill the tank in y hours, then time taken by pipe B alone to fill the tank is $\frac{xy}{y-x}$ hours.
- If a pipe A can fill a tank in x hours and a pipe B can empty the full tank in y hours (where y > x), then net part filled in 1 hour is $\frac{y-x}{xy}$.
- If a pipe A can fill a tank in x hours and a pipe B can empty the full tank in y hours (where x > y), then net part emptied in 1 hour $\frac{x-y}{xy}$.
- Three pipes A, B and C can fill a tank in x y, and z hours respectively. If all the three pipes are opened simultaneously, then time taken by all the pipes to fill it is $\frac{xyz}{xy+yz+zx}$ hours.
- If two pipes are filing a tank at the rate of x hours and y hours respectively and a third pipe is emptying it at the rate of z hours, then in one hour the part of the tank filled is $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$ .Time taken to fill the tank is $\frac{yz+zx-xy}{xyz}$.

SHORT ANSWER TYPE QUESTIONS

1. A cistern is filled by Pipe A and Pipe B together in 2.4 hours. Pipe A alone can fill the cistern at the rate of 100 litres per hour. Pipe B alone can fill the cistern in 4 hours. What is the capacity of the cistern?
2. A cistern can be filled in 6 hours by taps P and Q. If tap R also joins them, then cistern is filled in 5 hours. Tap P can fill the cistern at twice the rate of tap Q. In what time tap Q and R fill the cistern?
3. A cistern is normally filled with water in 10 hours but takes 5 hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty the cistern in
4. A pipe can fill a tank in 6 hours. Another pipe can empty the tank in 12 hours. If both pipes are opened simultaneously, the part of tank filled by both pipes in 1 hour is?
5. A tap having diameter 'd' can empty a tank in 84 min. How long another tap having diameter '2d' take to empty the same tank?
6. A pump can fill a tank with water in 20 minutes and another pump can fill the same tank m 30 minutes. If both the pumps are opened together, then how much time will be taken to fll the tank completely?
7. Two pipes A and B together can fill a tank in 10 minutes. If pipe A takes 15 minutes less than B to fill the tank alone, then find the time taken by pipe B to fill the tank alone.

8. Pipes A and B can fill a tank in 6 hours and 8 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, find the time in which the tank can be filled.
9. A pipe can fill a tank in 40 minutes. Due to a leakage in the bottom it took 60 minutes to fill the tank. How much time will it take for the leakage to empty the full tank?
10. Two pipes A and B can fill a tank in ½ hour and 1 hour respectively. A pipe C can empty the tank in 2 hours. If all three pipes are opened simultaneously, how long does it take to fill the empty tank?

ANSWERS

1. 600 litres
2. 11.25 hours
3. 30 hours
4. **1/12 th part**
5. 21 minutes
6. 12 minutes
7. 30 minutes
8. 4 hours
9. 2 hours
10. 24 minutes

LONG ANSWER TYPE QUESTIONS:

1. A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 minutes after the pipes A and B are opened, find when the cistern becomes full?
2. Two pipes A and B can fill a tank in 30 minutes and 45 minutes respectively. Both pipes A and B are opened together for some time and then pipe B is turned off. If the tank is filed in 20 minutes, then find after how many minutes the pipe B is turned of ?
3. A tap can fill a cistern in 8 hours and another tap can empty it in 16 hours. Find the time taken to fill the tank, if both the taps are open.
4. A cistern has three pipes A B, and C . A and B can fill it in 3 hours and 4 hours respectively while C can empty the completely filled cistern in 1 hour. If the pipes are opened in order at 3 P.M., 4 P.M. and 5 P.M. respectively, at what time will the cistern be empty ?
5. A tap can fill a tank in 40 minutes and another outlet pipe can empty it in 24 minutes. If the tank is already two-fifths full and both the taps are opened together, will the tank be filled or emptied? How long will it take before the tank is either filled completely or emptied completely, as the case may be?
6. A cistern can be filled by an inlet pipe in 20 hours and can be emptied by an outiet p 25 hours. Both the pipes are opened. After 10 hours, the outlet pipe is closed, find the total time taken fill the cistern.
7. Three pipes A, B and C are installed to fill a tank. Pipes A and B opened together can fill the tank in the same time in which C can alone fill the tank. If pipe B can fill the tank 15 minutes faster than pipe A and 5 minutes slower than pipe C, then find the time required by pipe A to fill the tank alone.
8. Two pipes can fill a tank in 20 minutes and 24 minutes respectively and a waste pipe can empty 3 gallons of water per minute. If all the three pipes working together can fill the tank in 15 minutes, find the capacity of the tank.
9. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find the time required by first pipe to fill the tank alone.
10. Three pipes A, B and C can fill a tank in 72 minutes. If all the three pipes remain opened tor 36 minutes and then pipe C is closed it took 1 hour more to fill the tank by pipes A and B. Find the time required to fill the tank by pipe C alone.

ANSWERS

1. 7 min 30 sec.
2. 15 minutes
3. 16 hours
4. 7 : 12 pm
5. Empty in 24 minutes
6. 28 hours
7. 30 minutes
8. 120 gallons
9. 15 hours
10. 3 hours

Races and Games:

- Suppose X and Y are participating in a race.
  "X gives Y a start of x metres" means Y starts the race x metres ahead of X.
  "X gives Y a start of t-minutes" means X will start t minutes after B starts the race.
  "X beats Y by x metres" means when X reach the finishing point, Y is x metres behind X.
  "X beats Y by t minutes" means Y finish t minutes after X.
  "X beats Y by x metres and t minutes" means Y is x-metres behind X and finish race t-minutes after X.
- A game of 100 means that the person among the participants who scores 100 points first is the winner.
  " X beats Y by 25 points" or "X can give Y 25 points" means X scores 100 while Y scores only 75points (100-25).

SHORT ANSWER TYPE QUESTIONS

1. In a 100 m race, A can give B 10 m and C 28 m. In the same race B can give C.
2. A and B take part in 100 m race. A runs at 5 kmph. A gives B a start of 8 m and still beats him by 8 seconds. Find the speed of B
3. In a 500 m race, the ratio of the speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by
4. In a 100 m race, A beats B by 10 m and C by 13 m. In a race of 180 m, B will beat C by
5. In 100 m race, A covers the distance in 36 seconds and B in 45 seconds. In this race A beats B by
6. In a game of 100 points, A can give B 20 points and C 28 points. Then, B can give C
7. A can run 22.5 m while B runs 25 m. In a kilometre race B beats A by
8. In a 300 m race A beats B by 22.5 m or 6 seconds. B's time over the course is
9. In a 50 m race, A can give a start of 5 m to B and a start of 14 m to C. In the same race, how much start can B give to C?
10. In a 500 m race, A reaches the final point in 28 s and B reaches in 35 s. By how much distance does A beat B?

ANSWERS

1. 20 metres
2. 4.14 km/h
3. 20 m
4. 6 m
5. 20 m
6. 10 points
7. 100 metres
8. 80 seconds
9. 10 m
10. 100 m

LONG ANSWER TYPE QUESTIONS

1. In a 10 km race. A, B, and C, each running at uniform speed, get the gold, silver, and bronze medals, respectively. If A beats B by 1 km and B beats C by 1 km, then by how many metres does A beat C?
2. A and B run a km race A wins by 1 minute. A and C run a km race and A wins by 375 metres. B and C run a km race and B win By 30 seconds. Find the time that C takes to run a km race.
3. Two men A and B walk around a circle 1500 metres in circumference. A walks at the rate of 140 metres, and B at the rate 80 metres per minute .if they start from the same point, and walk in the same direction, when will they be together again?
4. In a race of 200 metres, A can give a start of 10 metres to B, C give a start of 20 metres to B. The start that C can give to A, in the same race is

ANSWERS

1. 1900 m
2. 4 minutes
3. 25 minutes
4. 30 metres

Numerical Inequalities:

- Any two real number associated by $<, >, \leq\ or\ \geq$ forma numerical inequality
- If a, b are positive numbers and AM and GM are their arithmetic mean and geometric mean respectively, then $\boldsymbol{AM = \frac{a+b}{2}}$ and $\boldsymbol{GM = \sqrt{ab}}$

$$AM - GM = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} = \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2} \geq 0$$

$$\therefore \boldsymbol{AM \geq GM}$$

- Properties of Linear Inequalities

  For three real numbers $a, b\ and\ c$
  - $If\ a > b, and\ b > c\ then\ a > c$
  - $If\ a > b, then\ a \pm\ c\ >\ b \pm c$
  - $If\ a > b, and\ c > 0\ (c\ is\ positive)\ then\ ac > bc.$
  - $If\ a > b, and\ c < 0\ (c\ is\ negative)\ then\ ac < bc$ $(Inequality\ Changes)$
  - $If\ a > b,\ then\ \frac{1}{a} < \frac{1}{b}$ $(Inequality\ Changes)$

SHORT ANSWER TYPE QUESTIONS

1. Insert appropriate sign of inequality:

$$\sqrt{3}(\sqrt{50}+\sqrt{32})\_\_\_\_\_\_\_\_\_\_\_ 2\sqrt{54}+3\sqrt{24}$$

2. Insert appropriate sign of inequality:

$$(\sqrt{3}+\sqrt{8})\_\_\_\_\_\_\_\_\_\_\_\sqrt{90}$$

3. Solve the inequality for real x:

$$4 - 2x \geq 3x + 19$$

4. Solve the following inequality, and graph the solution on number line.

$$2x - 5 \leq x + 2 < 3x + 8$$

5. $Covishield\ vaccine$ need to be stored and transported at $+2°\ to + 8°\ Celsius$ temperature. What is the range of temperature in degree Fahrenheit if the conversion is given by $C = \frac{5}{9}(F - 32)$ where C and F are temperature in degree Celsius and degree Fahrenheit respectively? Is it safe to transport the vaccine in a temperature-controlled truck, which maintains temperature between40°F to 45°F?

ANSWERS

1. < $\boldsymbol{9\sqrt{6} < 12\sqrt{6}}$
2. > $\sqrt{3}+\sqrt{8} \geq 2\sqrt{24} = \sqrt{96} > \sqrt{90} \Rightarrow \boldsymbol{\sqrt{3}+\sqrt{8} > \sqrt{90}}$
3. $\boldsymbol{x \leq -3\ OR\ (-\infty, -3]}$
4. $\boldsymbol{-3 < x \leq 7; (-3, 7]}$
5. $2 < C = \frac{5}{9}(F - 32) < 8 \Rightarrow \boldsymbol{35.6 < F < 46.4.}$

   Yes, it’s safe to transport the vaccine by truck.

LONG ANSWER TYPE QUESTIONS

1. $If\ a, b, c\ are\ positive\ real\ numbers, then\ find\ the\ least\ value\ of\ (a+b)(b+c)(c+a).$
2. How many litres of water should be added to 120 litres of 60% solution of acid so that the resulting mixture will contain more than 20% but less than 30% of acid content?
3. How many litres of 25% solution of acid, should be added to 600 litres of 10% solution of acid so that the resulting mixture will contain more than 12% but less than 15% of acid content?

ANSWER

1. **8abc**

$$AM \geq GM \implies \quad a+b \geq 2\sqrt{ab}, \quad b+c \geq 2\sqrt{bc} \ \ and \quad c+a \geq 2\sqrt{ca}$$

$$\implies (a+b)(b+c)(c+a) \geq 2\sqrt{ab}.2\sqrt{bc}.2\sqrt{ca} = \mathbf{8abc}$$

2. Let $x\ litres$ of water is added to 120 litres of solution, then the total mixture is $(x+120)\ litres$.

$$20\%\ of\ (x+120) < 60\%\ of\ 120 < 30\%\ of\ (x+120)$$

$$\implies \frac{20}{100}(x+120) < \frac{60}{100} \times 120 < \frac{30}{100}(x+120)$$

$$\implies 2(x+120) < 6 \times 120 < 3(x+120)$$

$$\implies 2x+240 < 720 < 3x+360$$

$$\implies 2x+240 < 720\ and\ 720 < 3x+360$$

$$\implies 2x < 480\ and\ 360 < 3x$$

$$\implies x < 240\ and\ 120 < x\ ie.\ \mathbf{120 < x < 240}$$

$Hence\ the\ quantity\ of\ water\ should\ be\ more\ than\ 120\ litres\ but\ less\ than\ 240\ litres.$

3. Let $x\ litres$ of 25% solution of acid is added to 600 litres of solution, then the total mixture is $(x+600)\ litres$.

$$12\%\ of\ (x+600) < 25\%\ of\ x + 10\%\ of\ 600 < 15\%\ of\ (x+600)$$

$$\implies \frac{12}{100}(x+600) < \frac{25x}{100} + \frac{10}{100} \times 600 < \frac{15}{100}(x+600)$$

$$\implies 12(x+600) < 25x+6000 < 15(x+600)$$

$$\implies 12x+7200 < 25x+6000 < 15x+9000$$

$$\implies 12x+7200 < 25x+6000\ and\ 25x+6000 < 15x+9000$$

$$\implies 1200 < 13x\ and\ 10x < 3000$$

$$\implies \frac{1200}{13} < x\ and\ x < 300\ ie.\ \mathbf{\frac{1200}{13} \approx 92.31 < x < 300}$$

$$Hence\ the\ quantity\ of\ water\ should\ be\ more\ than\ \frac{1200}{13}\ litres\ but\ less\ than\ 3000\ litres.$$

MATRICES

SOME IMPORTANT RESULTS/CONCEPTS

1. **Definition:** A matrix is a rectangular arrangement of numbers into rows and columns.
2. **Order of a Matrix**: $(No.of\ rows) \times (No.of\ Columns)$
3. **Types of Matrices**:

3.1 **Row Matrix**: A matrix with only one row (ie. Order = $1 \times n$).

3.2 **Column Matrix:** A matrix with only one column (ie. Order = $m \times 1$.

3.3 **Square Matrix:** A matrix with number of rows equal to number of column (m=n).

3.4 **Upper Triangular Matrix:** A square matrix with $a_{ij} = 0\ \forall\ i < j$.

3.5 **Lower Triangular matrix:** A square matrix with $a_{ij} = 0\ \forall\ i > j$

3.6 **Diagonal Matrix:** A square matrix with all non-diagonal elements equal to zero.($a_{ij} = 0\ \forall\ i \neq j$).

3.7 **Scalar Matrix:** A diagonal Matrix with all diagonal elements equal. $(a_{ii} = k)$

3.8 **Identity Matrix:** A diagonal Matrix with all diagonal elements equal to 1. $(a_{ii} = 1)$

3.9 **Zero, Null Matrix**: A matrix will all elements equal to zero. $(a_{ij} = 0\ \forall\ i,j)$

4. **Equality & Operation of Matrices**:

4.1 **Equal Matrix**: $A\ matrix\ A = (a_{ij})and\ B = (b_{ij})are\ said\ to\ be\ equal\ iff$

$$Order\ of\ A = Order\ of\ B\ \&\ a_{ij} = b_{ij} \forall i,j$$

4.2 **Addition & Subtraction of Matrix**: If matrices A=(a_{ij}) and B=(b_{ij}) are of order $m \times n$, then the sum/difference of matrices $A \pm B = C = (c_{ij})$ is also of order $m \times n$ where $c_{ij} = a_{ij} \pm b_{ij}$

4.3 **Multiplication of Matrices:** If matrix A=(a_{ij}) is of order $m \times n$ and B=(b_{ij}) is of order $n \times p$, then the product of matrices AB = C =(c_{ij}) is of order $m \times p$ where $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

5. **Transpose of a Matrix**: The matrix obtained by interchanging its row and columns.
   The transpose of matrix $A_{m \times n}$ =(a_{ij}) is given by A^t_{mxn}= (a_{ji}).
   Properties: (i)$(A')' = A$ (ii) $(A \pm B)' = A' \pm B'$ (iii) $(AB)' = B'A'$

6. **Symmetric & Skew-Symmetric Matrix**
   A square matrix A = (a_{ij}) is said to be symmetric matrix if $a_{ij} = a_{ji}$. $(ie. A' = A)$
   A square matrix A = (a_{ij}) is said to be skew-symmetric matrix if $a_{ij} = -a_{ji}$. $(ie. A' = -A)$
   *The diagonal elements of a skew symmetric matrix are Zero.$(a_{ii} = 0)$
   Any matrix A can be written as sum of a symmetric matrix P and skew symmetric matrix Q, where

$$P = \frac{A + A'}{2} \; and \; Q = \frac{A - A'}{2} ; A = P + Q$$

SHORT ANSWER TYPE QUESTIONS

1. If the order of the matrix is m×n, then how many elements will there be in the matrix?
2. What is the order of the matrix A= $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & 0 \end{bmatrix}$?
3. **Find the total number of possible matrices of order 2 × 3 with each entry -1, 0 or 1.**
4. If matrix A is of order 2 ×3 and matrix B is of order 3 ×2 , then what is the order of matrix AB?
5. Find the values of a,b,c and d if $\begin{bmatrix} a & -2 \\ b & 7 \end{bmatrix} = \begin{bmatrix} 2 & c \\ 3 & 2c + d \end{bmatrix}$
6. Find x and y such that 2 $\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$
7. If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, then find the matrix M.
8. If A= $\begin{bmatrix} 0 & 5 & -3 \\ -5 & 0 & a \\ b & 2 & 0 \end{bmatrix}$ is a skew symmetric matrix of order 3, then find the values of a and b.
9. If A = $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ then find $.AA'$
10. If A is any square matrix of order n then $(A+A')^I$ is

ANSWERS

1. mn
2. 2 x 3
3. $3^6 = 729$
4. 2 x 2
5. a=2, b=3, c=−2,d=11
6. x=2, y=−8
7. $M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$
8. $a = -2, b = 3$
9. $AA' = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
10. Symmetric Matrix

LONG ANSWER TYPE QUESTIONS

1. Find x and y, if $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$
2. If the matrix X=$\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ is equal to the matrix Y=$\begin{bmatrix} p - q & 2p + r \\ 2p - q & 3r + s \end{bmatrix}$ then find value p,q,r and s
3. Express the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.
4. If A = $\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$,then verify $(A + B)' = A' + B'$
5. If A=$\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$and B=$\begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$,verify that $(AB)' = B'A'$

ANSWERS

1. x = 3, y=2
2. p=1,q=2,r=3,s=4. 3. P = $\frac{1}{2}\begin{bmatrix}2 & 5 & 7\\5 & -4 & 3\\7 & 3 & 2\end{bmatrix}$, Q=$\frac{1}{2}\begin{bmatrix}0 & -1 & -1\\1 & 0 & -1\\1 & 1 & 0\end{bmatrix}$

DETERMINANTS

SOME IMPORTANT RESULTS/CONCEPTS

1. Determinant of a square matrix A is a real number associated with the matrix.

   1.1 $If\ A = [a]\ then\ |A| = a$

   1.2 $If\ A = \begin{bmatrix}a & b\\c & d\end{bmatrix}, then\ |A| = \begin{vmatrix}a & b\\c & d\end{vmatrix} = ad - cb$

   1.3 $If\ A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix} then$

$$|A| = \begin{vmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{vmatrix} = a_{11}\begin{vmatrix}a_{22} & a_{23}\\a_{32} & a_{33}\end{vmatrix} - a_{12}\begin{vmatrix}a_{21} & a_{23}\\a_{31} & a_{33}\end{vmatrix} + a_{13}\begin{vmatrix}a_{21} & a_{22}\\a_{31} & a_{32}\end{vmatrix}$$

2. **Minor and Cofactor**

   Minor of an element is the determinant obtained by deleting the row and column of the element. The order of minor is one less than the order of the matrix. It is represented by M_{ij}.

   Co-factor of an element $a_{ij} = C_{ij} = (-1)^{i+j}M_{ij}$

3. **Adjoint of** $\boldsymbol{A} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\ is\ \begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

4. $\boldsymbol{Adjoint\ of\ a\ matrix\ A}\ = Adj.A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\C_{21} & C_{22} & C_{23}\\C_{31} & C_{32} & C_{33}\end{bmatrix}' = \begin{bmatrix}C_{11} & C_{21} & C_{31}\\C_{12} & C_{22} & C_{32}\\C_{13} & C_{23} & C_{33}\end{bmatrix}$

5. $\boldsymbol{Inverse\ of\ a\ matrix\ A} = A^{-1} = \frac{1}{|A|}Adj.A = \frac{1}{|A|}\begin{bmatrix}C_{11} & C_{21} & C_{31}\\C_{12} & C_{22} & C_{32}\\C_{13} & C_{23} & C_{33}\end{bmatrix}$

6. For a square matrix A of order n,

   6.1 $|kA| = k^n|A|$

   6.2 $|AB| = |A|.|B|$

   6.3 $|Adj.A| = |A|^{n-1}$

   6.4 $|A^{-1}| = \frac{1}{|A|}$

7. **Solution of System of equations:**

   7.1 **Matrix Method**: Let the system of equation is given by

$$\begin{matrix}a_1x + b_1y + c_1z = d_1\\a_2x + b_2y + c_2z = d_2\\a_3x + b_3y + c_3z = d_3\end{matrix} \Rightarrow \begin{bmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix} \Rightarrow AX = B$$

$$\boldsymbol{X = A^{-1}B}$$

   7.2 **Cramer's Rule**: Let the system of equation is given by

$$\begin{matrix}a_1x + b_1y + c_1z = d_1\\a_2x + b_2y + c_2z = d_2\\a_3x + b_3y + c_3z = d_3\end{matrix} \Rightarrow \begin{bmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix} \Rightarrow AX = B$$

Compute $\Delta = \begin{vmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{vmatrix}, \Delta_x = \begin{vmatrix}d_1 & b_1 & c_1\\d_2 & b_2 & c_2\\d_3 & b_3 & c_3\end{vmatrix}, \Delta_y = \begin{vmatrix}a_1 & d_1 & c_1\\a_2 & d_2 & c_2\\a_3 & d_3 & c_3\end{vmatrix}, \Delta_z = \begin{vmatrix}a_1 & b_1 & d_1\\a_2 & b_2 & d_2\\a_3 & b_3 & d_3\end{vmatrix}$

$$\boldsymbol{x} = \frac{\Delta_x}{\Delta}, \quad \boldsymbol{y} = \frac{\Delta_y}{\Delta}, \quad \boldsymbol{z} = \frac{\Delta_z}{\Delta}$$

SHORT ANSWER TYPE QUESTIONS

1. If $A = [-5]$ $then\ |A| =?$
2. If $A = \begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix}$ $then\ |A| =?$
3. If A = $\begin{bmatrix} 3 & 2 & -4 \\ 6 & -1 & -2 \\ -7 & 1 & 2 \end{bmatrix}$ $then\ find\ the$ cofactor of a_{23}.
4. A is a scalar matrix with scalar k ≠ 0 of order 3 then find A^{-1}.
5. The square matrices A, B, C, D and E are of same order and I is an identity matrix .If the product $ABCDE = I$ then find C^{-1} .
6. A and B are two square matrices of same order. If AB = B^{-1} then find A^{-1}.
7. If A is a matrix of order 3 and $|A| = -7$then find the $|Adj.A|$.
8. If A is a matrix of order 3 and $|A| = -5$then find the $|-2A|$.
9. If A is a matrix of order 4 and $|A| = -3$ then find the $|A^{-1}|$.
10. If A is a matrix of order 3 and $|A| = -2$then find the $|A \times Adj(A)|$.

ANSWERS

1. -5
2. -1
3. -17
4. $\frac{1}{k}I$
5. DEAB
6. B^2
7. 49
8. -40
9. $\frac{-1}{3}$
10. -8

LONG ANSWER TYPE QUESTIONS

1. For the matrix A=$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$,Show that $A^2 - 4A + 7I = 0$. Hence find A^{-1}.
2. Find the inverse of a matrix A= $\begin{bmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{bmatrix}$.
3. Find the inverse of a matrix A= $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$
4. Solve the following system of equations.

$$5x + 2y - 7z = 33$$
$$3x - 2y + 4z = -3$$
$$x - y + z = -1$$

5. Rajiv, Rahul and Raj went for shopping vegetables. Rajiv brought 500gm tomato, 2 kg potatoes and 1 kg apples. Rahul brought 1kg tomato, 1 kg potatoes and 2 kg apples. Raj brought 1 kg tomatoes, 2 kg potatoes only. Rajiv, Rahul and Raj paid Rs.200, Rs.310 and Rs.140 respectively. Find the cost of 1 kg tomatoes, 1 kg potatoes and 1 kg apples.

ANSWERS

1. $\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$
2. $\frac{1}{25}\begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$
3. $\begin{bmatrix} -2 & 4/5 & 9/5 \\ 3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix}$
4. $x = 3, y = 2, z = -2$
5. 1 kg tomatoes = Rs.80
   1 kg potatoes = Rs.30
   1 kg apples = Rs.100

DIFFERENTIATION AND ITS APPLICATIONS

SOME IMPORTANT RESULTS/CONCEPTS

INCREASING AND DECREASING FUNCTIONS:

Increasing Function: Function f(x) is increasing on (a, b) if

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \quad for\ all\ x_1, x_2 \in (a,b)$

Decreasing Function: Function f(x) is decreasing on (a, b) if

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2) \quad for\ all\ x_1, x_2 \in (a,b)$

Critical Points are the points on the curves where curve changes its direction i.e. tangent to the curve on these points are parallel to x-axis.

Monotone Function is the function which is either increasing or decreasing in whole of its domain i.e. no critical point exists on it.

Algorithm to solve the problems:

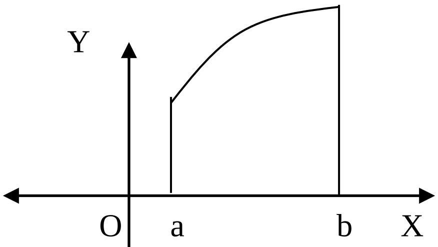
(1)To show y = f(x) is increasing or decreasing on (a, b), find $f'(x)$ & show that for increasing function $f'(x) > 0$ and for decreasing function $f'(x) < 0$

(2) To find intervals in which y = f(x) is increasing or decreasing in (a, b)

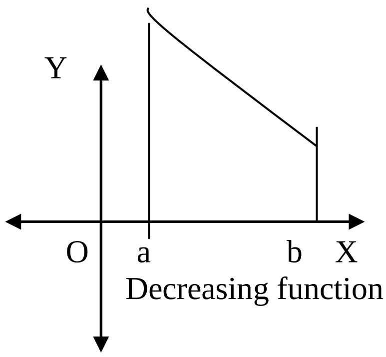
(i) Find $f'(x)$; (ii) put $f'(x) = 0$ to find critical points like x_1 & x_2 *etc.*

(iii) With critical points construct intervals of increasing and decreasing e.g. $(a, x_1), (x_1, x_2), (x_2, b)$

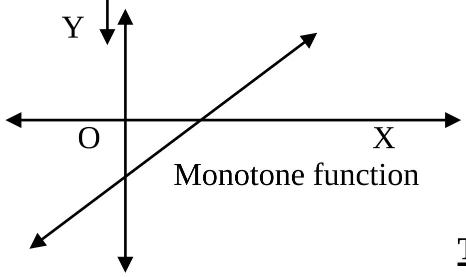
(iv) For each interval check value of $f'(x)$ by substituting values from the intervals. If $f'(x) > 0$ function is increasing on the interval and if $f'(x) < 0$ function is decreasing on the interval.



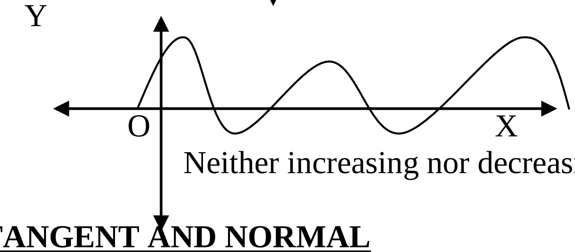
Increasing function



Decreasing function



Monotone function



Neither increasing nor decreasing function

TANGENT AND NORMAL

Slope of a line is denoted by m and defined as m = $\tan\theta$ here θ is angle inclination of the line.

Points to remember:

1. Slope of a line parallel to X-axis is 0.

2. Slope of a line parallel to Y-axis is ∞ i.e. $\frac{1}{0}$

3. Slope of a line equally inclined to the axes is ± 1.

4. Two lines having slopes m_1 & m_2 are

(i) Parallel if $m_1 = m_2$ (ii) perpendicular if $m_1.m_2 = -1$

5. Slope of a line ax + by + c = o is m = $-\frac{a}{b}$.

6. Slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$

Slope of the tangent to the curve y = f(x) at the point A (x_1, y_1) lying on it is $m = (dy/dx)_A$

Also slope of normal is given by m = -1 / (slope of tangent)

Equation of tangent and normal to the curve y = f(x) at the point A (x_1, y_1) on the curve

$$y - y_1 = m(x - x_1)$$

MAXIMUM AND MINIMUM

FIRST DERIVATIVE TEST

Minimum: Function f(x) attains local minimum value at x = a if there exist a neighborhood (a - h, a + h) such that $f'(a-h) < 0$ and $f'(a+h) > 0$

Maximum: Function f(x) attains local maximum value at x = a if there exist a neighborhood (a - h, a + h) such that $f'(a-h) > 0$ and $f'(a+h) < 0$

SECOND DERIVATIVE TEST

Minimum: Function f(x) attains local minimum value at x = a if $f''(x) > 0$

Maximum: Function f(x) attains local maximum value at x = a if $f''(x) < 0$

Algorithm to solve the problems: (i) Find $f'(x)$; (ii) put $f'(x)$= 0 to find points of maximum and minimum; (iii) find $f''(x)$; (iv) Use second derivative test to find points of max. and min.; (v) Substitute points of max./min. in f(x) to get local max/min values.

APPLICATIONS OF DERIVATIVES IN COMMERCE AND ECONOMICS:

COST FUNCTION: If c is the total cost incurred in producing and marketing x units of a certain commodity, then a function relating C and x is called a cost function.

Fixed cost: It is the sum of all costs that are independent of the level of production.

Variable cost: It is the sum of all costs that are dependent on the level of production.

Supply function: A relation between the price per unit and the quantity supplied by the producer in the market at that price is called the supply function.

Total Revenue Function: If R is the total revenue collected by a company when it sells x units of a product at price p per unit , then R is given by R = px

Profit function : If R(x) is the total revenue received and c(x) is the total cost incurred in the production of x units of a commodity, then the function p(x) given by p(x) = R(x)- C(x).

Break –Even Point: The breakeven point is the level production where the revenue from sale is equal to the cost of production and marketing.

Average cost: It is the cost of producing and marketing each unit of the product.

Marginal cost: It is defined as the rate of change of the total cost with respect to each unit of product.

Average Revenue: Revenue per unit is known as average revenue.

Marginal Revenue: It indicates the rate at which the total revenue changes with respect to units sold.

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the interval in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is decreasing
2. Find the interval in which the function $f(x) = e^{2x}$ is increasing.

3. Find the interval in which $f(x) = x^2 e^{-x}$ is increasing.

4. Let *f* have second derivative at *c* such that $f'(c) = 0$ and $f''(c) > 0$, then *c* is a point of ______
5. Find the value of 'a' for which f (x) = x^3 – ax is an increasing function of R.
6. Find the value of 'b' for which f (x) = x^2 + b x + 1 is an increasing on [1, 2].
7. Show that function $f(x) = x^3 - x^2 + 7x + 3$ do not have any maximum or minimum value.
8. Find two positive numbers whose sum is 15 and the sum of whose square is minimum.
9. Show that among the rectangle of given perimeter, the square has the greatest area.
10. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of *a* .

ANSWERS

1. [-2,3] 2. All real numbers R 3. (0, 2) 4. Local Minima 5. $a \le 0$ 6. $b \ge -2$ 8. 9, 6
10. a = 52

SHORT ANSWER TYPE QUESTIONS

1. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$, is an increasing function throughout its domain.
2. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.
3. Prove that the function $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in **R.**
4. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at x=e.
5. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top by cutting off squares from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.?
6. Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope 2/3.
7. Find the equation of tangent to the curve $y^2 = 4ax$ at $(at^2, 2at)$
8. Find the points on the curve $y = x^3$ at which slope of the tangent is equal to the ordinate of the point.
9. If the line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$ find the value of *m*.
10. Prove that the curves $x = y^2$ *and* $xy = k$ cut at right angles if $8k^2 = 1$
11. For manufacturing a certain item the fixed cost is Rs 9000 and the variable cost of producing each unit is Rs 30. The average cost of producing 60 units is
   (a)Rs 150 (b) Rs 180 (c) Rs 240 (d) Rs 120

(a) If the selling price of a commodity is fixed at Rs 45 and the cost function is C(x) = 30x+240, then the break even point is X=10 (b) 12 (c) 15 (d) 16

12. The demand function of a monopolist is given by x= 100-4p .The quantity at which the marginal revenue will be zero
   (a) 25 (b) 10 (c) 50 (d) 40
13. If the total cost function is given by c(x)= 10x-7x^2 +3x^3 , then marginal average cost is given by

(a) 10-14x +9x^2 (b) 10-7x +3x^2 (c) -7+6x (d) -14+18x

14. If the demand function for a product is p= $\frac{80-x}{4}$, where x is the number of units and p is the price per unit , then the value of x for which the revenue will be maximum is
(a) 40 (b) 20 (c) 10 (d) 80

15. A company has fixed cost of Rs 26000 . The cost of producing one item is Rs 30. If this item sells for Rs 43 , find the break-even point.

ANSWERS

5. 5 6. (3, 2) 7. $t\,y - x = a\,t^2$ 8. (0,0),(3,27) 9. m =1 11. (c) Rs 240 12. (d) 16 13. (c) 50 14. (c) -7+6x 15. (a)40 16. 2000 items

LONG ANSWER TYPE QUESTIONS

1. Find the absolute maximum and minimum values of a function *f* given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5].

2. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank?

3. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

4. The sum of the perimeter of a circle and square is *k*, where *k* is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

5. Show that the height of cylinder of maximum volume that can be inscribed in a sphere of radius 'a' is $\frac{2a}{\sqrt{3}}$.

6. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

7. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}(1/3)$.

8. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of base.

9. Show the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$

10. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit max light through the whole opening.

11. The cost function for a certain commodity is $C(x) = 12 + 3x - \frac{1}{3}x^2$. *Write down total cost* Fixed cost, variable cost and average cost when 3 units are produced.

12. A company is selling a certain product. The demand function for the product is linear. The company can sell 2000 units when the price is Rs. 8 per unit and it can sell 3000 units when the price is Rs. 4 per unit. Determine: (i) the demand function (ii) the total revenue function

13. The price of selling one unit of a product when x units are demanded is given by the equation $p = 4000 - 2x$. The fixed costs of product are Rs. 20000 and Rs. 1484 per unit as the cost of production.Find the level of sales at which the company can expect to cover its costs.

14. For a monopolist's product , the demand function is $p = \frac{50}{\sqrt{x}}$ and average cost function AC=$0.5 + \frac{2000}{x}$.Find the profit maximising level of output. At this level , show that the marginal revenue and marginal cost are equal.
15. If $C = ax^2 + bx + c$ represent the total cost function ,find : (i) Slope of average cost curve (ii) slope of marginal cost curve
16. A manufacturer produces computers data storage floppies at the rate ox x units per week and his total cost of production and marketing is $C(x) = \frac{x^3}{3} - 10x^2 + 15x + 30$.Find the optimal number of floppies produced per week at which the marginal cost and average variable cost attain their respective minima.
17. There are 60 newly built apartments. At a rent of Rs. 450 per month all will occupied. However, one apartment will be vacant for each Rs. 15 increase in rent. An occupied apartment requires Rs 60 per month for maintenance. Find the relationship between the profit and number of unoccupied apartments. What is the number of vacant apartments for which profit is maximum?

ANSWERS:

1. absolute maximum value 56, absolute minimum value 24 2. Rs. 1000

10. Length = $\frac{20}{\pi+4}m$, $breadth$= $\frac{10}{\pi+4}m$

11. TC =Rs. 18,FC= Rs. 12 ,VC =Rs 6, AC= Rs 6

12(i) p= $16 - \frac{x}{250}$ (ii) R(x)= $16x - \frac{x^2}{250}$

13. Breakpoint points are x= 8, 1250.Hence, the company must produce at least 8 units to cover its cost.

14. P(x) is maximum when x=2500, MR=MC when x =2500

15. (i) $a - \frac{c}{x^2}$ (ii) 2a

16. AVC is minimum when 15 floppies are produced and marketed.

17. Maximum profit is obtained when 17 apartments remain vacant.

INTEGRATION AND ITS APPLICATIONS

SOME IMPORTANT RESULTS/CONCEPTS

INDEFINITE INTEGRALS

* $\int x^n dx = \frac{x^{n+1}}{n+1} + C$	* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\| + C$, if $x > a$
* $\int 1.dx = x + C$	* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\| + C$, if $x > a$
* $\int \frac{1}{x^n} dx = -\frac{1}{x^n} + C$	* $\int \frac{dx}{\sqrt{a^2+x^2}} = \log \| x + \sqrt{x^2+a^2} \| + C$
* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$	* $\int \frac{dx}{\sqrt{x^2-a^2}} = \log \| x + \sqrt{x^2-a^2} \| + C$
* $\int \frac{1}{x} dx = \log_e x + C$	* $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left\|x + \sqrt{x^2+a^2}\right\| + C$
* $\int e^x dx = e^x + C$	* $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left\|x + \sqrt{x^2-a^2}\right\| + C$
* $\int a^x dx = \frac{a^x}{\log_e a} + C$	* $\int \{f_1(x) \pm f_2(x) \pm ....f_n(x)\}dx$ $= \int f_1(x)dx \pm \int f_2(x)dx \pm .... \pm \int f_n(x)dx$
* $\int u.v\,dx = u.\int v.dx - \int\left[\int v.dx\right]\frac{du}{dx}.dx$	
* $\int \lambda f(x)dx = \lambda\int f(x)dx + C$	

DEFINITE INTEGRALS

SOME IMPORTANT RESULTS/CONCEPTS

* $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x) = \int f(x)\,dx$,
* $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$
* $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
* $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
* $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
* $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
* $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$
* $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$
* Cost Function, $C(x) = \int MC(x)dx$ where MC is Marginal Cost
* Revenue Function, $R(x) = \int MR(x)dx$ where MR is Marginal Revenue
* Consumers' Surplus, CS = $\int_0^{x_0} f(x)dx - p_0x_0$ where f(x) is the demand curve
* Producers' Surplus, PS = $p_0x_0 - \int_0^{x_0} g(x)dx$, where g(x) is the supply curve
* The equilibrium price is the price where the amount of the product that consumers want to buy (quantity demanded) is equal to the amount producers want to sell (quantity supplied). This mutually desired amount is called the equilibrium quantity.

Indefinite Integration

Short Answer Type Questions

Q1. $\int \frac{1}{x^{\frac{3}{2}}}dx$ Q2. $\int \log_x x\,dx$ Q3. $\int \frac{e^{log\sqrt{x}}}{x}dx$ Q4. $\int \frac{e^{6logx}-e^{5logx}}{e^{4logx}-e^{3logx}}dx$ Q5. $\int (1-x)\sqrt{x}dx$

Q6. $\int \frac{x^3-x^2+x-1}{x-1}dx$ Q7. $\int (\sqrt{x}-\frac{1}{\sqrt{x}})^2\,dx$ Q8. $\int \frac{(x+1)(x-2)}{\sqrt{x}}dx$ Q9. $\int \frac{1}{\sqrt{(x+1)}+\sqrt{x}}dx$

10. $\int \frac{1}{\sqrt{(x+3)}-\sqrt{(x+2)}}dx$ Q11. $\int (ax+b)^2\,dx$ Q12. $\int x\sqrt{x+2}\,dx$

Q13. $\int \frac{1}{(2-7x)^4}dx$ Q14. $\int e^{3x+2}\,dx$ Q15. $\int (e^x+e^{-x})^2\,dx$ Q16. $\int (x+3)\sqrt{x+2}\,dx$

Q17. $\int \frac{(x-1)}{\sqrt{x+1}}dx$ Q18. $\int \frac{1}{(e^x+1)(e^{-x}+1)}dx$ Q19. $\int x^2e^{x^3}\,dx$ Q20. $\int x^5(1+x^6)^{\frac{3}{2}}dx$

Q21. $\int \frac{(1+\sqrt{x})^n)}{\sqrt{x}}dx$ Q22. $\int \frac{x+1}{x^2+2x+2}dx$ Q23. $\int \frac{1}{x(1+logx)}dx$ Q24. $\int \frac{(x+1)(x+logx)^2}{x}dx$

Q25. $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}dx$ Q26. $\int x^3e^{x^2}\,dx$ Q27. $\int logx\,dx$ Q28. $\int x^2e^x\,dx$

Q29. $\int x^n logx\,dx$ Q30. $\int \frac{log(logx)}{x}dx$

Answers

Q1. $\frac{-2}{\sqrt{x}}+c$ Q2. $x+c$ Q3. $2\sqrt{x}+c$ Q4. $\frac{x^3}{3}+c$ Q5. $\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+c$ Q6. $\frac{x^3}{3}+x+c$

Q7.$\frac{x^2}{2} + logx - 2x + c$ Q8. $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4\sqrt{x}$+c Q9. $\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$

Q10. $\frac{2}{3}(x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$ Q11. $(ax+b)^3$/3+c Q12. $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$
Q13.$\frac{1}{21(2-7x)^3} + c$ Q14.$\frac{e^{2x+3}}{3} + c$ Q15.$\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x + c$ Q16. $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$
Q17. $\frac{2}{3}(x+1)^{\frac{3}{2}} - 4(x+1)^{\frac{1}{2}} + c$ Q18. $\frac{-1}{(e^x+1)} + c$ Q19.$\frac{e^{x^3}}{3} + c$ Q20.$\frac{1}{15}(1+x^6)^{\frac{5}{2}} + c$
Q21.$\frac{2(1+\sqrt{x})^{n+1}}{(n+1)} + c$ Q22.$\frac{1}{2}log(x^2+2x+2) + c$ Q23. $log(1+logx) + c$ Q24. $\frac{(x+logx)^3}{3} + c$ Q25. $\log(e^x - e^{-x}) + c$ Q26.$\frac{1}{2}(x^2-1)e^{x^2} + c$ Q27.$xlogx - x + c$ Q28.$(x^2-2x+2)e^x + c$
Q29.$\frac{x^{n+1}}{n+1}logx - \frac{x^{n+1}}{(n+1)^2} + c$ 30. $logx(log(logx) - 1) + c$

Long Answer Type Questions:

Q1.$\int \frac{1}{(x-1)(x+3)}dx$ Q2. $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$ Q3.$\int \frac{1-x}{x(1-2x)}dx$ Q4. $\int \frac{5x+4}{(x^2-1)(x+2)}dx$
Q5. $\int \frac{3x-2}{(x+1)(x-2)^2}dx$ Q6.$\int \frac{2x}{(x^2+1)+(x^2+2)}dx$ Q7.$\int \frac{3x+2}{x^2+5x-6}dx$ Q8.$\int \frac{xe^x}{(x+1)^2}dx$ 9. $\int \frac{1}{e^x-1}dx$
Q10.$\int \frac{(x-3)}{(x-1)^3}e^x\, dx$

Answers:

Q1.$\frac{1}{4}log(x-1) - \frac{1}{4}log(x+3) + c$ Q2. $\frac{-1}{6}log(x-1) - \frac{1}{3}log(x+2) + \frac{1}{2}log(x-3) + c$

Q3.$logx - \frac{1}{2}log(1-2x) + c$ Q4. $\frac{3}{2}log(x-1) + \frac{1}{2}log(x+1) - 2log(x+2) + c$ Q5. $\frac{-5}{9}log(x+1) + \frac{5}{9}log(x-2) - \frac{4}{3(x-2)} + c$ Q6.$log\left(\frac{(x^2+1)}{(x^2+2)}\right) + c$ Q7. $\frac{5}{7}(x-1) + \frac{16}{7}log(x+6) + c$
Q8.$\frac{e^x}{(x+1)} + c$ Q9.$log\frac{e^x-1}{e^x} + c$ Q10.$\frac{e^x}{(x-1)^2} + c$

Definite Integral

Short Answer type

Q1. $\int_3^4 \frac{2x+3}{x^2+3x+2}dx$ Q2.$\int_0^1 xe^x\, dx$ Q3.$\int_1^4 (\sqrt{x} + \frac{1}{\sqrt{x}})\, dx$ Q4.$\int_0^1 \frac{1}{\sqrt{x+1}-\sqrt{x}}dx$ 5.$\int_1^2 xlogx\, dx$
Q6.$\int_e^{e^2} \frac{1}{xlogx}dx$ Q7.$\int_0^1 x\sqrt{1-x^2}\, dx$ Q8.$\int_5^7 \frac{logx}{x}dx$ Q9.$\int_0^1 e^x\sqrt{1+e^x}\, dx$ 10.$\int_1^\infty \frac{1}{x(1+x^2)}dx$

Answers

Q1.$log\frac{3}{2}$ Q2. 1 Q3. $\frac{63}{8}$ Q4.$\frac{4\sqrt{2}}{3}$ Q5.$2log2 - \frac{3}{4}$ Q6.log2 Q7.$\frac{1}{3}$ Q8.$\frac{1}{2}log\frac{7}{5}log35$
Q9.$\frac{2}{3}(1+e)^{\frac{3}{2}} - \frac{4}{3}\sqrt{2}$ Q10.$\frac{1}{2}log2$

Properties Based Definite Integral

Q1.$\int_{-1}^1 \frac{e^x}{e^x+e^{-x}}dx$ Q2.$\int_0^1 \frac{logx}{logx+log(1-x)}dx$ Q3.$\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ Q4.$\int_0^1 x(1-x)^n\, dx$
Q5.$\int_1^4 |x-5|\, dx$ Q6. $\int_0^a \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$ Q7. $\int_2^8 |x-5|\, dx$ Q8.$\int_{-1}^1 log\frac{(1-x)}{(1+x)}dx$

Q9. $\int_0^1 x\sqrt{1-x}\,dx$ Q10.if $[x]$ stands for integral part of x then show that $\int_0^1 [5x]\,dx = 2$

Answers

Q1. 1 Q2. $\frac{1}{2}$ Q3. $\frac{1}{2}$ Q4. $\frac{1}{(n+1)(n+2)}$ Q5. $\frac{15}{2}$ Q6. $\frac{a}{2}$ Q7. 9 Q8. 0 Q9. $\frac{4}{15}$

INTEGRATION AND ITS APPLICATIONS

SHORT ANSWER TYPE QUESTIONS

1. Find the consumers' surplus for the demand function $p = 25 - x - x^2$ when $p_0 = 19$.
2. The demand function for a commodity is $p = \frac{10}{x+1}$. Find the consumers' surplus when the prevailing market price is 5
3. The supply function for a commodity is $p = x^2 + 4x + 5$ where x denotes supply. Find the producers' surplus when the price is 10.
4. If the demand function for a commodity is $p = 25 - x^2$, find the consumers' surplus for $p_0 = 9$.
5. If the supply function is $p = 3x^2 + 10$ and $x_0 = 4$, find the producers' surplus.
6. The marginal cost of production of x units of a commodity is 30+2x. It is known that fixed costs are Rs.120. Find the total cost of producing 100 units.
7. The marginal revenue function of a commodity is $MR = 7 - \frac{6}{(x+2)^2}$,find the revenue function. Also find the revenue obtained on selling 4 units of the product.
8. Determine the cost of increasing output from 100 to 200 units if the marginal cost $MC = 0.003x^2 - 0.01x + 2.5$.
9. A manufacturer's marginal cost function is $\frac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units.
10. The marginal revenue function of a commodity is $MR = 9+6x^2$, find the revenue function. Also, find the revenue obtained on selling 5 units of the product.

ANSWERS

1. $\frac{22}{3}$ 2. 10 log2 – 5 3. $\frac{8}{3}$ 4. $\frac{128}{3}$

5. 128 6. $C(x) = 120+30x+x^2$ 7. $R(x) = 7x + \frac{6}{x+2} - 3$ 8. Rs. 7100

9. $500\sqrt{2x+25}$ 10. $R(x)=9x+2x^3$

LONG ANSWER TYPE QUESTIONS:

1. The demand and supply functions are p= 50-8x and p = 5+x respectively. Determine the consumer's surplus and producer's surplus at equilibrium price.
2. The demand and supply functions are $p_d = 25 - x^2$ and $p_s = 2x+1$ respectively. Determine the consumer's surplus and producer's surplus at equilibrium price.
3. For a particular commodity the demand function $D(x) = 26 - \frac{x^2}{100}$ and the supply function is $S(x) = \frac{x^2}{400} + 6$. Determine to the nearest rupee the consumer's surplus and producer's surplus if the market is at equilibrium.

4. Find the consumer's surplus and producer's surplus under pure competition for demand function p = $\frac{8}{x+1} - 2$ and supply function p = $\frac{1}{2}$(x+3) where p is the price and x is the quantity.
5.Suppose that demand is given by the equation x_d = 500-50p where x_d is quantity demanded and p is the price of the good. Supply is described by the equation x_s= 50+25p where x_s is quantity supplied. What is the equilibrium price and quantity?
6.If the demand and supply curve for the computers is D = 100-6p , S = 28+3p respectively where p is the price of the commodity. What is the quantity of computers bought and sold at equilibrium?
7. The marginal cost of product in x units of a commodity in a day is given as MC = 16x – 1591. The selling price is fixed at Rs.9 per unit and the fixed cost is Rs.1800 per day. Determine
(i) Cost function (ii) Revenue function (iii) Profit function (iv) Maximum profit that can be obtained in a day.
8.Suppose when x units of a commodity are produced, the demand is p(x) = 58 $-x^2$ rupees per unit and the marginal cost is MC = 6+$\frac{1}{4}x^2$. Assume that there is no overhead. Find the(i)total revenue and marginal revenue (ii) the value of x that maximizes profit
9.The marginal cost and marginal revenue of a firm are given as MC= $3x^2 - 118x + 1315$ and MR =1000 – 4x. Determine the maximized profit using definite integrals, assuming fixed cost is Rs 595.
10. The marginal cost and marginal revenue of a firm are given as MC= 14+x and MR =36-24x+3x^2. Determine the maximized profit using definite integrals, assuming fixed cost is zero.

ANSWERS

1.CS= $\frac{128}{3}$, PS = 16 2.CS= $\frac{16}{3}$, PS = $\frac{32}{3}$ 3. CS= 427, PS = 107 4. CS= 8 log 2 $-$4, PS = $\frac{1}{4}$
5. p= 6 and x = 200 6. p = 8, S = 52
7.(i)C(x) = $8x^2 - 1591x + 1800$ (ii)R(x) = 9x (iii) P = -$8x^2 + 1600x - 1800$ (iv) ₹78,200
8.(i) R = $58x - x^3$ MR = 58$-3x^2$ (ii) x = 4 9. 15,330. 10. 10.5

DIFFERENTIAL EQUATIONS AND MODELING

SOME IMPORTANT RESULTS/CONCEPTS

DIFFERENTIAL EQUATION

An equation involving derivative(s) of the dependent variable with respect to the independent variable(s) is called a differential equation.

A differential equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.

$\frac{dy}{dx} + \frac{y}{x} = x^3$ …… (i) $\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^2 = 0$ …… (ii)

ORDER OF A DIFFERENTIAL EQUATION: The order of differential equation is defined as the highest ordered derivative of the dependent variable with respect to the independent variable involved in the differential equation.

DEGREE OF A DIFFERENTIAL EQUATION

For the degree of a differential equation to be defined it must be a polynomial equation in its derivatives. The degree of a differential equation, when it is a polynomial equation in its derivatives is the highest power (positive integral index) of the highest order derivative involved in the differential equation.

DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELING: Differential equations play a pivotal role in modern world ranging from, engineering to ecology and from economics to biology.

Many real-world problems involve rate of change of a quantity. These problems can be described by mathematical equations. In many natural phenomenon and real life applications a quantity changes at a rate proportional to the amount present.

GROWTH AND DECAY MODELS The mathematical model for exponential growth or decay is given by f(t) = Ae^{kt} or y= Ae^{kt}

Where: **t** represents time , **A** the original amount y or f(t) represents the quantity at time t ;k is a constant that depends on the rate of growth or decay If k > 0, the formula represents exponential growth ; If k < 0, the formula represents exponential decay

SHORT ANSWER TYPE QUESTIONS

1. Find the order and degree of D.E. $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$
2. Find the product of order and degree of D.E. $\left(\frac{dy}{dx}\right)^4+3y\frac{d^2y}{dx^2}=0$
3. Find the (a + b) where a and b are order and degree of D.E. $(y''')^2+(y'')^3+(y')^4+y^5=0$ Respectively.
4. Find the number of arbitrary constants in general solution of above 3 questions.
5. Find the number of arbitrary constants in particular solution of above 3 questions.
6. Verify that y= ae^{-x} $\textit{is a solution of}$ $\frac{dy}{dx}+y=0$.
7. Check whether y = (a+bx) e^{2x} is a solution of $y_2-4y_1+4y=0$?
8. Find the solution of differential equation $\frac{dy}{dx}=(e^x+1)\,y$.
9. Find the solution of differential equation $x^5\frac{dy}{dx}=-y^5$
10. Find the solution of differential equation $\frac{dy}{dx}=\frac{x+1}{2-y}$
11. Find the differential equation of the family of circles having centre at origin.
12. Form the differential equation of the family of circles having centre on y-axis and passing through origin.
13. Solve the differential equation: $\frac{dy}{dx}+y=1$
14. Solve the differential equation: $ylogy\,dx-xdy=0$
15. Solve the differential equation $\frac{dy}{dx}=e^{x+y}+x^2e^y$

LONG ANSWER TYPE QUESTIONS

1. In a certain culture of bacteria, the number of bacteria increased 5 times in 10 hours. How long did it take for the number of bacteria to double?
2. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?
3. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25,000 in the year 2004, what will be the population of the village in 2009?
4. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}\,hours$?
5. Find the particular solution of the differential equation $\frac{dy}{dx}=x(2\,log\,x+1)\,given\ that\ y=0\ when\ x=2$

ANSWERS: -

SHORT ANSWER TYPE

1.2,1 2. 2 3. 5 4. 2,2,35. 0,0,0 6.yes 7. Yes 8. ln|y|= $e^x + x + c$ 9. $x^4 + y^4 = cx^4y^4$

10. $x^2 + y^2 + 2x - 4y + c = 0$ 11. X+4y_1=0 12. $(x^2 - y^2)y_1 - 2xy = 0$

13. $y = ke^{-x} + 1 \; where \; e^c = k$ 14. $y = e^{cx}$ 15. $e^x + e^{-y} + \frac{x^3}{3} = c$

LONG ANSWER TYPE

1. 4.3 hours 2. $\frac{2\log 2}{log\left(\frac{11}{10}\right)}$ hours 3. 31250 4. 604.9 (appx) 5 . $y = x^2 \log x - 4\log 2$

CASE STUDY

A rumour on WhatsApp spreads in a population in 5000 people at a rate proportional to the product of the number of people who have heard it and the number of people who have not. Also, it is given the 100 people initiate the rumour and a total of 500 people know the rumour after 2 days.

Based on the above information, answer the following questions

(1) If y (t) denote the number of people who know the rumour at an instant t, then what is the maximize value of y(t).
(2) What will be the value of y (2)?
(3) What will be the value of y at any time instant t?

ANSWERS

(i) 5000 (ii) 500 (iii) $y = \frac{5000}{49e^{-5000kt}+1}$

PROBABILITY DISTRIBUTIONS

SOME IMPORTANT RESULTS/CONCEPTS

PROBABILITY/ BINOMIAL DISTRIBUTION

1. Then the mathematical expectation is the weighted average of the possible values of X given by E(X) = $x_1 p_1 + x_2 p_2 + x_3 p_3, \ldots + x_n p_n = \sum_{i=1}^{n}(x_i p_i)$

2.Let μ = E (X) be the mean of X. Then the variance of X, denoted by Var (X) is given by Var(X) = $\sum_{i=1}^{n}(x_i^2 p_i) - [\sum_{i=1}^{n}(x_i p_i)]^2$

3.The standard deviation denoted by $\sigma_x = \sqrt{Var\ (X)}$

4. Probability of 'r' successes in 'n' Bernoulli trials is given byP ('r' successes) = $C_r^n\ p^r q^{n-r}$,Where n = number of trials

r = number of successful trials = 0, 1, 2, 3, …, n
p = probability of a success in a trial
q = probability of a failure in a trial
And, p + q = 1
5. In a binomial distribution having 'n' number of Bernoulli trials where p denotes the probability of success and q denotes the probability of failure, then
i. Mean = np
ii. Variance = npq
iii. Standard Deviation =$\sqrt{npq}$

POISSON DISTRIBUTION

Let X be the discrete random variable which represents the number occurrence of events over a period of time. If X follows the Poisson distribution, then the probability of occurrence of 'k' number of events over a period of time is given by
P(X =k) = f (k) =$\frac{\lambda^k e^{-\lambda}}{k!}$
Where e is Euler's number (e = 2.71828…)
'k' is the number of occurrences of the event such that k = 0, 1,2, …
And λ = E(X) = Var(X) = np , is a positive real number

NORMAL DISTRIBUTION

1.A continuous random variable X is defined in terms of its probability density function also known as PDF as well
2. A continuous random variable X is designed to follow normal distribution with constant parameters = E(X) and Var(X) = σ^2 and written as X∼ N (μ , σ^2)
f(x) ≥ 0 ∀ x ∈(- ∞. ∞) , where y = f(x) is probability density function of X
such that f(x) =$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ ------------- (i)
where μ ∈(- ∞. ∞) is the mean of normal distribution and is the standard deviation
3. When a random variable can take on any value within a given range where the probability distribution is continuous, it is called a normal distribution .
4. The mean, median and mode of the sample space are exactly the same.
5. In a normal distribution curve the total area below the curve is always equal to 1 unit;
i.e.$\int_{-\infty}^{\infty} f(x)$ =1
6. In a standard normal distribution of data, the Z-score is given by Z =$\frac{x-\mu}{\sigma}$
7. The standard normal curve also represents the probability. We calculate three types of probabilities:
Case (i)P(Z< z) = F(z) Case (ii)P(z_1<Z< z_2) = F(z_2) – F(z_1) Case (iii) P(Z > z) = 1 - F(z)
The value of F(z) are given in the form of a standard normal distribution table

BINOMIAL DISTRIBUTION

SHORT ANSWER TYPE QUESTIONS

1.In a binomial distribution, the probability of getting success is ¼ and standard deviation is 3. Calculate its mean.
2. A random variable X has the following probability function

X	0	1	2	3	4	5	6	7
P(X)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2+a$

Find P(X< 3)
3. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.
4. What is the probability of guessing at least 8 correct answers out of 10 true/false question.

5. A box contains 100 bulbs of which 10 are defective. Find the probability that out of a sample of 5 bulbs drawn one by one with replacement none is defective.

6. A random variable 'X' has the following probability distribution:

X	0	1	2	3
P(X)	K	K	2K	K

Find the value of P(X=1) + P(X=2)+ P(X=3)

7. In a binomial distribution, the sum of its mean and variance is 1 & 8. Find the probability of two successes if the event was conducted 5 times.

8. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then find the value of its parameter p.

9.The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X=x) = \begin{cases} k & ,if\ x=0 \\ 2k & ,if\ x=1 \\ 3k & ,otherwise \end{cases}$$

(i) Determine the value of k (ii) Find $P(X \le 1)$

10.Find the probability distribution of X , the number of heads in two tosses of a coin (or a simultaneous toss of two coins)

11. For the following probability distribution:

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

Find the value of E(X).

12. What is the mean of the numbers obtained on throwing a die having written 1on three faces, 2 on two faces and 5 on one face?

13. For Binomial distribution B (4, 2/3) , find the value of variance.

ANSWERS

1. 6 2.$\frac{3}{10}$ 3. 1- $\left(\frac{9}{10}\right)^{10}$ 4. $\frac{7}{128}$ 5. $\left(\frac{9}{10}\right)^{5}$ 6. 0.8 7.$\frac{128}{625}$

8.p = $\frac{2}{3}$ 9.(i) k =1/6 (ii) ½ 10.P(X=0)=1/4, P(X=1)=1/2, P(X=2)= ¼ 11. -1.8

12. 2 13.0.89

LONG ANSWER TYPE QUESTIONS

1. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	a	4a	3a	7a	8a	10a	6a	9a

   (i) Determine the value of a.
   (ii) Find P(X<3), P(X>=4), P(0<X<5)

2. A coin is tossed twice (or two coins are tossed simultaneously).
   (i) Find the probability distribution of X, the number of heads.
   (ii) If a random variable Y is defined on this sample space as the number of tails, then find the probability distribution of Y .

3. Four bad oranges are mixed accidentally with 16 oranges. Find the probability distribution of the number of bad oranges in a draw of two oranges.

4. Three balls are drawn one by one without replacement from a bag containing 5 white and 4 red balls. Find the probability distribution of the number of white balls drawn.
5. A random variable X has the probability distribution:

X	1	2	3	4	5	6	7	8
P(X)	.15	.23	.12	.1	.2	.08	.07	.05

If events E= { X is a prime number} and F= { X<4}, then find P (EUF)
6. Suppose 2% of the people on the average are left handed. Find the probability of 3 or more left handed among 100 people.
7. Six dices are thrown 729 times. How many times do you expect at least three dice to show a five or six?
8. A die is thrown thrice. A success is 1 or 6 in a throw. Find the mean and variance of the number of successes
9. The sum of mean and variance of a binomial distribution is 15 and the sum of their squares is 117. Determine the value of n .

ANSWERS

1. (i) 1/48 (ii) P(X<3)=1/6, P(X>=4)=33/48, P(0<X<5)=11/24

2. (i)

X	0	1	2
P(X)	1/4	1/2	1/4

(ii)

X	0	1	2
P(X)	1/4	1/2	1/4

3.

X	0	1	2
P(X)	12/19	32/95	3/95

4.

X	0	1	2	3
P(X)	1/21	5/14	10/21	5/42

5. .78 6.0.325 7.233 8. Mean=1 , variance= 2/3 9. n= 27

POISSON DISTRIBUTION

SHORT ANSWER TYPE QUESTIONS

1. For a poisson's Distribution, find P(2), given $\lambda = 1$
2. If 2% $of\ the$ books bound at a certain workshop have defective binding, find the probability that 5 books out of 400 books will have defective binding.
3. Suppose that X has a Poisson distribution. If P(X=2)= $\frac{2}{3}$P(X=1),evaluate P(X=0).
4. If X has a Poisson distribution such that P(x=1)=P(x=2) and e^{-2}=0.1353, Find P(X = 4).
5. Jobs arrive at a factory at an average rate of 5 in an 8 hours shift. The arrival of the jobs following Poissions distribution. The average service time of job on the factory is 40 minutes. The service time follows exponential distribution. What will be the Ideal time (in hours) at the factory per shift?
6. If X is a Poisson variable such that P(X=1)=2P(X=2), then find P(X=0) .

ANSWERS

1. 0.184 2.0.0928 3.$e^{\frac{-4}{3}}$ 4. 0.0902
5. 14/3 6. e^{-1}

LONG ANSWER TYPE QUESTIONS

1.Suppose that a manufactured product has 2 defects per unit of product inspected. using Poisson distribution, calculate the probabilities of finding a product (1) without any defect (2) 3 defect 93) 4 defect

ANSWERS

1. (1) 0.135 (2) 0.180 (3) 0.09

NORMAL DISTRIBUTION

SHORT ANSWER TYPE QUESTIONS

1. If Z is a standard normal variable then P(0 < Z < 1.7) = ...
2. What is the range of normal distribution ?
3. The mean of a distribution is 60 with standard deviation 5. Assuming that the distribution is normal, what percentage of items be between 65 and 75.
4. The monthly salaries of workers in a certain factory are normally distributed. The mean salary is Rs. 4000 and standard deviation is Rs. 450. If 668 workers are getting salary less than Rs. 3325, find the total number of workers in the factory.
5. If X is a normal distribution random variable with mean μ=10 and standard deviation σ=2, Then what is the value of P(X< 13) ?
6. If X is normally distributed with mean 20 and standard deviation 4, then what is the value of standard normal variable Z corresponding to X = 21 ?
7. Given that mean of a normal variate X is 12 and standard deviation is 4, then find the Z-Score of data point 20.
8. Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, what is the probability that a candidate who takes the test will score between 90 and 110?
9. The mean of a distribution is 60 with standard deviation 5. Assuming that the distribution is normal, what percentage of items be between 65 and 75?

ANSWERS

1. F(1.7) – F(0) 2. (−∞ to ∞) 3. 15.74% 4. 10000 5. 0.93
6.0.25 7.2 8. 0.6826 9. 15.74%

LONG ANSWER TYPE QUESTIONS

1. Suppose the temperature during June is normally distributed with mean $20^0 C$ and standard deviation 3.33^0C. Find the probability that the temperature is between 21.11^0C and 26.66^0C.
2. If X is a normal variable with mean 11 and standard deviation 1.5, find the number m such that P(X>m)=0.09.

3. . In a sample of 1000 items, the mean weight is 450kg with a standard deviation 15 Kg. Assuming the normality of the distribution, find the number of items weighing between 40 and 60 Kg.
4. 1000 light bulbs with a mean life of 120 days are installed in a new factory; their length of life is normally distributed with standard deviation 20 days. How many bulbs will expire in less than 90 days?

ANSWERS

1.0.3479 2.m= 13.01 3. 471 4.67

INFERENTIAL STATISTICS

SOME IMPORTANT RESULTS/CONCEPTS

SHORT ANSWER TYPE QUESTIONS

1. The records of weights of the male population follows the normal distribution. Its mean and standard deviations are 70 kg and 15 kg respectively. If researcher considers the records of 50 males , then what would be the mean and standard deviation of the chosen sample ?
2. A machine makes car wheels and in a random sample of 26 wheels , the test statistics is found to be 3.07. As per the t-distribution test (of 5% level of significance) , what can you say about the quality of wheels produced by the machine ? (use $t_{25}(0.05) = 2.06$)
3. For the purpose of t-test of significance , a random sample of size (n) 34 is drawn from a normal population , then what is the degree of freedom (v) ?
4. In a school , a random sample of 145 students is taken to check whether a students average calory intake is 1500 or not . The collected data of averages calories intake of sample students is presented in a frequency distribution , what is it called ?
5. If a classroom has seating capacity of 30 students , then what will be degree of freedom for the students ?
6. In a factory producing 2000 electric lamps per day , 20 lamps are picked up randomly to check the quality by the quality control department. What do you call to no of lamps chosen for inspection ?
7. What do you called to measurable characteristics of a population ?
8. If you have selected a sample of Indian Cricket team fans and found that 75% of your sample are male ,describe the type of sample ?
9. If a TV rating service dials numbers for obtaining sample of households ,then which type of sample is represented by this ?
10. A news channel conducts pre election survey and predicts that the candidate A will get 30% of the vote . According to news channel the survey had margin of error of 5%. What will be the confidence interval for the survey ?
11. A 95% confidence interval for a population mean was reported to be 152 to 160 . If $\sigma = 15$, what sample size was used in this study ?
12. A company producing steel tubes and in a random sample of 10 tubes the test statistics is 0.476. As per the t-distribution test (of 5% level of significance), What can you say about the quality of steel tubes ? (Given t_9 (0.05)= 2.262)
13. What do you called assumed hypothesis which is tested for rejection considering it to be true ?
14. A population consists of four observations 1,3,5,7. What is variance ?
15. A sample of 50 bulbs is taken at random. Out of 50 we found 15 bulbs are of Bajaj, 17 are of Surya and 18 are of Crompton . What is the point estimate of population proportion of Surya ?

ANSWER KEY

1. Mean = 70 and standard deviation = 2.12	2. n=26 $\|t\|$ =3.07 > $t_{25}(0.05)$= 2.06 Inferior quality
3. n=34 ,v= 34-1 =33	4. sampling distribution

5. n=30 , v=30-1 =29	6. sample
7. parameter	8. representative sample
9. unrepresentative sample	10. 25% - 35%
11. $\bar{x}$ be sample mean , E be margin error , then $\bar{x} - E = 152$ and $\bar{x} + E = 160$ Then sample size =54	12. n=10 ItI =0.476 < $t_9(0.05)$= 2.262 superior quality
13. Null hypothesis	14. 5
15. 0.34	

LONG ANSWER TYPE QUESTIONS

Q.1 Consider the following hypothesis test: H_O: $\mu \leq 12$ H_a: $\mu > 12$, A sample of 25 provided a sample mean $\bar{x}$= 14 and a sample standard deviation S = 4.32

(i) Compute the value of the test statistic.

(ii) Use the t-distribution table to compute a range for the p-value.

(iii) At α = 0.05 what is your conclusion?

Q2 Suppose that a 95% confidence interval states that population mean is greater than 100 and less than 300.How would you interpret this statement?

Q3 A shoe maker company produces a specific model of shoes having 15 months average lifetime. One of the employees in their R and D division claims to have developed a product that last longer. This latest product was worn by 30 people and lasted on average for 17 months. The variability of the original shoe is estimated based on standard deviation of the new group which is 5.5 months. Is the designer's claim of a better shoe supported by the findings of the trial? Make your decision using two tailed test using a level of significance of $p < 0.05$

Q4 An electric light bulbs manufacturer claims that the average life of their bulb is 2000 hours. A random sample of bulbs is tested and the life (x) in hours recorded. The following were the outcomes: $\sum x = 127808$ and $\sum(\bar{x} - x)^2 = 9694.6$. Is there sufficient evidence, at the 1% level, that the manufacturer is over estimating the life span of light bulbs?

Q5 A fertilizer company packs the bags labeled 50kg and claims that the mean mass of bags is 50 kg with a standard deviation 1 kg. An inspector points out doubt on its weight and tests 60 bags. As a result he finds that mean mass is 49.6 kg. Is the inspector right in his suspicions?

Q 6 The average heart rate for Indians is 72 beats/minute. To lower their heart rate , a group of 25 people participated in an aerobics exercise programme. The group was tested after six months to see if the group had significantly slowed their heart rate. The average heart rate for the group was 69 beats/minute with a standard deviation of 6.5 .Was the aerobics program effective in lowering heart rate?

ANSWERS

Q1. Given n =25, $\bar{x}$=14,S=4.32, μ_O =12 & $t = \frac{\bar{x} - \mu_O}{S/\sqrt{n}}$ = 10/4.38 = 2.31

t=2.31 and degrees of freedom = 25-1=24.

(ii) t = 2.31 > 0

So, p-value of 2.31 = Area under the t-distribution curve to the right of t. From the t-distribution table, we find that t = 2.31 lies between 2.064 and 2.492 for which area lies between 0.01 and 0.025, so p-value lies between 0.01 and 0.025. since 0.01 <p-value < 0.025

(iii) Given α = 0.05 Since p-value <0.05 So, reject Ho.

Q2 Confidence interval lies between 100 and 300. So, Sample mean ($\bar{x}$) lies between 100 and 300

As μ=$\bar{x} \pm E$, 100=$\bar{x} + E$ and 300=$\bar{x} - E$ Solving $\bar{x} = 200$

Q3 H_O: $\mu = 15$ & H_a: $\mu \neq 15$ Given μ_O =15 ,n=30 $\bar{x} = 17$ S=5.5

$t = \frac{\bar{x}-\mu_O}{S/\sqrt{n-1}}$ $= \frac{17-15}{5.5/\sqrt{30-1}}$ =1.953 As α = 0.05 , α/2= 0.025 $t_{\alpha/2}$=2.045 As 1.953<2.045
i.e. t< $t_{\alpha/2}$ So null hypothesis is accepted.

Q4 Given $\mu_O = 2000$, $\sum x = 127808$ and $\sum(\bar{x}-x)^2 = 9694.6$.

$\bar{x}=\frac{\sum x}{n}$ => $n=\frac{\sum x}{\bar{x}}$ $=\frac{127800}{2000}$=63.904≈ 64 α =1% =0.01, S = $\sqrt{\frac{\sum(\bar{x}-x)^2}{n-1}}$= $\sqrt{\frac{9694.6.}{63}}$ =12.4

t=$\frac{\bar{x}-\mu_O}{S/\sqrt{n-1}}$ .As sample mean is not given so the given data is not sufficient to reject null hypothesis.

Q 5 Given μ_O =50 ,n=60 $\bar{x} = 49.6$ S=1 , t=$\frac{\bar{x}-\mu_O}{S/\sqrt{n-1}}$=$\frac{49.6-50}{\frac{1}{\sqrt{60-1}}}$=-3

α =0.05(by default) So As α = 0.05 & α/2= 0.025 , $t_{\alpha/2}$=2.001 df=60-1 =59

For two tail test to reject null hypothesis we should have t≤ $t_{\alpha/2}$, i.e. -3≤ -2.001 null hypothesis is rejected.
So the inspector is right.

Q6 Given μ_O =72 n=25 $\bar{x} = 69$ S=6.5 So As α = 0.05, α/2= 0.025, $t_{\alpha/2}$=2.064 df =25-1 =24

t=$\frac{\bar{x}-\mu_O}{S/\sqrt{n-1}}$=$\frac{69-72}{\frac{6.5}{\sqrt{25-1}}}$=-2.261 (For two tail test to reject null hypothesis we should have t≤ $t_{\alpha/2}$)

i.e. -2.261≤ -2.064 (null hypothesis is rejected, So the aerobics program is efficient.)

TIME BASED DATA

SOME IMPORTANT RESULTS/CONCEPTS

1.A time series is a sequentially recorded numerical data points for a given variable arranged in a successive order to track variation
2.The purpose of time series is to show an increasing growth pattern over time for a variable.
3. When data of the variable is collected at distinct time intervals, for a specified period of time, it is called time series data.
4. When data for one or more variables is collected at the same point in time, it is called cross-sectional data.
5. When data is collected in a combination of time series data and cross-sectional data, it is called pooled data.
6. A time series in which data of only one variable is varying over time is called a univariate time series.
7. When a time series is a collection of data for multiple variables and how they are varying over time, it is called multivariate time series.
8. Secular trend component or simply called trend series, is the smooth, regular and long-term variations of the series, observed over a long period of time.
9. Seasonal component is a time series captures the periodic variability in the data, capturing the regular pattern of variability; within one-year periods.
10. Cyclical component is a time series shows an oscillatory movement where period of oscillation is more than a year where one complete period is called a cycle. The real Gross Domestics Product(GDP) provides good examples of a time series that displays cyclical behavior.
11. Irregular component is a time series in which fluctuations are unaccountable, unpredictable or sometimes caused by unforeseen circumstances like – floods, natural calamities, labor strike etc.
12. Trend can be measured using by the following methods:
i. Graphical method
ii. Semi averages method
iii. Moving averages method
iv. Method of least squares
13.Procedure of finding the equation of trend line
Method of least squares is a technique for finding the equation which best fits a given set of bservations. Suppose we are given n number of observations and it is required to fit a straight line to these data. Note that n, the number of observations can be odd or even.Recall that the general linear equation to represent a straight line is:

y = a + bx, --------------------- (i)
where y is the actual value, x is time; a and b are real numbers.
a = $\frac{\sum Y}{n}$ and b = $\frac{\sum XY}{\sum X^2}$

SHORT ANSWER TYPE QUESTIONS

1. The most commonly used mathematical method for measuring the trend is
(a) Semi Average (b)Moving Average (c)Free Hand Curve (d)Least Squares
2. In the measurement of the secular trend, the moving averages
(a) Smooth out the time series (b)Give the trend in a straight line
(b) Measure the seasonal variations (d) None of these
3. Multiplicative model for time series is O= . .
(a) T × S × C × I (b)T + S + C + I (c) T − S − C – I (d) None of these
4. In moving average method we cannot find trend values of some
(a) End Periods (b)Middle Period (c)Starting and End Periods (d) Starting Periods
5. A set of observations recorded at an equal interval of time is called
(a) Array data (b)Data (c)Geometric Series (d) Time series data

6. A rise in prices before Eid is an example of
(a) Cyclical Trend (b)Secular Trend (c) Irregular Trend (d) Seasonal Trend
7. Prosperity, Recession, and depression in a business is an exampleof
(a) Cyclical Trend (b)Secular Trend (c) Irregular Trend (d) Seasonal Trend
8. Graph of time series is called
(a) Line graph (b)Trend (c) Historigram (d) Histogram
9. A fire in a factory delaying production for some weeks is
(a) Cyclical Trend (b)Secular Trend (c) Irregular Trend (d) Seasonal Trend
10. Seasonal variations are
(a) Short term variation (b)Long term variation (c) Sudden variation (d) None
11. Time series data have a total number of components?
(a) 3 (b)6 (c)5 (d)4

ANSWERS

1. d 2. a 3.a 4.c 5.d 6.d 7.a 8.c 9.c 10.a 11.d

LONG ANSWER TYPE QUESTIONS

1. (i) Obtain the the three year moving averages from the following series of observations :

Year	1995	1996	1997	1998	1999	2000	2001	2002
	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4

(ii) Obtain five year moving average.

(iii) Construct also the 4 year centred moving average.

2. The average number , in lakhs, of working days lost in strikes during each year of the period (1981-90) : Calculate the three yearly moving average .

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

3. Fit a straight line trend by the method of least squares and tabulate the trend values from the following data

Year	2004	2005	2006	2007	2008	2009	2010
Sales (in thousands)	26	26	44	42	108	120	166

4. Fit a straight line trend by the method of least squares and tabulate the trend values from the following data

Year	2000	2001	2002	2003	2004	2005	2006
Production (in tones)	40	45	46	42	47	50	46

5. Fit a straight line trend by the method of least squares and tabulate the trend values from the following data

Year	1980	1981	1982	1983	1984	1985	1986	1987
Production (quintals)	346	411	392	512	626	640	611	796

6. Fit a straight line trend by the method of least squares and tabulate the trend values from the following data

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004
Y	4	7	7	8	9	11	13	14	17

7. Fit a straight line trend by the method of least squares and tabulate the trend values from the following data

Year	2005	2006	2007	2008	2009	2010
Profit	5	7	9	10	12	17

8. Find the three years moving average For the given values 15,24,18,33,42.

9. Assuming a four year cycle, calculate the trend by the method of Moving average from the following

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Value	12	25	39	54	70	87	105	100	82	65

10. Calculate the three year moving average of the following data

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

11. Given below are the consumer price index numbers (CPI) of the industrialsworkers.Find the best fitted trend line by the method of the least squares and tabulatedthe trend value

Year	2014	2015	2016	2017	2018	2019	2020
CPI	154	140	150	190	200	220	230

12. Given below are the data of workers Welfare expenses(In lakh Rs) in steel industries during 2001-2005. Use the method of the leastsquares to

(a) Tabulate the trend value

(b) Find the best fit for a straight-line trend (c) compute expected sale trend for year 2002

Year	2001	2002	2003	2004	2005
Welfare expenses(in lakh Rs)	160	185	220	300	510

13. Calculate the 5 yearly moving averages of the following time series of iron production:

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (in tonnes)	351	366	361	362	400	419	410	420	450	500

14. Calculate the 3 yearly moving averages from the following time series:

Year	2005	2006	2007	2008	2009	2010	2011	2012
Earnings: (Rs Lakhs)	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4

15. Calculate the 3 yearly moving averages of the following time series of cement production by a firm first 1 to 9 years :

Year	1	2	3	4	5	6	7	8	9
Production(in tonnes)	4	5	5	6	7	8	9	8	10

16. The profits of a soft drink firm in thousands of rupees during each month of a year were:

Months	Jan	Feb	March	April	May	june	July	Aug	Sept	Oct	Nov	Dec
Profit	1.2	.8	1.4	1.6	2.0	2.4	3.6	4.8	3.4	1.8	.8	1.2

Calculate the 3 monthly moving averages

17. In an influenza epidemic the number of cases diagnosed were:

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Numbers	2	0	5	12	20	27	46	30	31	18	11	5	0	1

Calculate 3-days moving averages.

18. The table given below shows the daily attendance in thousands at a certain conference over a period of two weeks:

Week 1	52	48	64	68	52	70	72
Week 2	55	47	61	65	58	75	81

Calculate the seven days moving averages.

19. The table below gives details of the electricity generated in million kilowatt hours in each quarter for the year 2018-2020.

Year	Quarter			
	FIRST	SECOND	THIRD	FOURTH
2018	8	7	6	9
2019	10	7	7	10
2020	11	7	8	10

Calculate the four quarterly moving averages.

20. Calculate five yearly moving averages of the number of students who have studied in a college given below:

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Number of students	442	427	467	502	512	515	520	527	515	541

21. Construct the 3- yearly moving averages from the following data:

Year	2003	2004	2005	2006	2007	2008	2009
Imported cotton consumption	129	131	106	91	95	84	93

22. Find out the secular trend by using four quarterly moving averages by using the data :

Year / Quarter	Q_1	Q_2	Q_3	Q_4
2018	29	37	43	34
2019	90	42	55	43
2020	47	51	63	53
2021	45	49	60	48

23. The number of road accidents in the city due to rash driving, over a period of 3 years, is given in the following table:

Year	Jan-Mar	Apr-Jun	Jul-Sept	Oct-Dec
2010	70	60	45	72
2011	79	56	46	84
2012	90	64	45	82

Calculate four quarterly moving averages.

24. Calculate the four yearly moving averages of the following time series of copper production (2010 to 2019):

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Production (in tonnes)	12	25	39	54	70	87	105	100	82	65

25. Daily absence from a school during 3 weeks is recorded as follows.Calculate 5 day moving averages.

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
WEEK 1	23	28	21	33	40
WEEK 2	38	52	43	58	63
WEEK 3	52	54	61	51	51

ANSWERS

1.(i) 4.067,4, 4.03, 4.40,4.40,3.73 (ii) 4,4.36,4.18,3.8 (iii) 4,4.2375,4.2652,4.025

2. 1.73, 1.97, 2.23, 2.83, 2.83, 4.1, 4.07, 5.13 3. y= 76+24x

Year	2004	2005	2006	2007	2008	2009	2010
Trend values	4	28	52	76	100	124	148

4. y= 45.143+1.036x

Year	2000	2001	2002	2003	2004	2005	2006
Trend values	42.03	43.07	44.11	45.14	46.18	47.21	48.25

5.y= 541.75+59.62x

Year	1980	1981	1982	1983	1984	1985	1986	1987
Trend values	333.08	392.7	452.32	511.94	571.56	631.18	690.8	750.42

6.y= 10+1.47x

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004
Trend values	4.12	5.59	7.06	8.53	10	11.47	12.94	14.41	15.88

7.y= 10+2.17x

Year	2005	2006	2007	2008	2009	2010
Trend values	4.58	6.74	8.91	11.09	13.26	15.43

8. 19, 25, 31 9. 39.75, 54.75, 70.75, 87.75, 92, 90.75

10. 3.667, 5.333, 6.667, 8.333, 10.333 , 11. Y = 152.1 + 16.6 X

Year	2014	2015	2016	2017	2018	2019	2020
Trend	102.3	118.9	135.5	152.1	168.7	185.3	201.9

12. Y=275+8.15x

Year	2004	2005	2006	2007	2008	2009	2010
Trend values	4	28	52	76	100	124	148

13. 368, 381.6, 390.4, 402.2, 419.8, 439.8
14. 4.067 , 4 , 4.03, 4.40, 4.40, 3.73
15. 4.67,5.33, 6, 7,,8,8.33,9
16. 1.4 ,1.7,2.1,2.8,3.4,3.5,3.1,2.3
17. 2.3,5.6,12.3,19.6,31.0,34.3,26.3,20.0,11.3,5.3,2.0
18. 60.86, 61.28,61.14, 60.17, 60.28, 61.14, 61.86, 63.14
19. 7.75,8,8.125,8.375,8.625,8.75,8.875,9
20. 470,484.6,503.2,515.2,517.8,523.6
21. --- , 122.00, 109.33, 97.33, 90.00, 90.66 , ---
22. 37.125,39.125,41.25,43.875,45.875,47.875,50.00,52.25,53.25,52.75,52.125,51.125
23. 62.875,63.5,63.125,64.75,67.625,70.00,70.875,70.5
24. 39.75, 54.75, 70.75, 84.75, 92, 90.75
25. 29,32,36.8,41.2,46.2,50.8,53.6,54,57.6,56.2,53.8

FINANCIAL MATHEMATICS

SOME IMPORTANT RESULTS/CONCEPTS

1. **(i) Rate of Return /Nominal Rate of Return/ Rate of interest**

$$= \frac{\textit{Current value of Investment} - \textit{cost of Investment}}{\textit{Cost of Investment}} \text{ X } 100\%$$

(ii) Rate of Return /Nominal Rate of Return/ Rate of interest = $\frac{D1+(P1-P0)}{P0}$ X 100%

Where *Po: price of shares/stocks at the time of Investment*

P1: price of S price of shares/stocks at the end of the year

D1: Dividend earned during the year

2. **Annual dividend earned** = No. of Shares x rate of Dividend x face value of one share
3. **Compound Annual Growth Rate**

$$\text{CAGR} = [\left(\frac{EV}{SV}\right)^{\frac{1}{n}} - 1] \text{ x } 100$$

EV = Investment's ending/Future value

SV = Investment's starting/Present value

n = Number of investment periods (year)

4. **Linear or straight-line method**:

$$D = \frac{C-S}{n}$$

Where D = the annual depreciation

C = the original cost of the asset

S = scrap value or salvage value

n = the useful life of asset in years

Remark: In the above formula, C-S is the total depreciation.

5. PERPETUITY: Perpetuity is an annuity where payments continue forever.

Amount of Perpetuity: Amount of perpetuity is undefined since it increases beyond all bounds as time goes on.

Present value of Perpetuity: We consider two types of perpetuity which are as follows:

(i) Present value of a perpetuity of R payable at the end of each period, the first being due one period hence37 $P = \frac{R}{i}$, where R = size of each payment, i = rate per period

(ii) Perpetuity of R payable at the beginning of each period, the first payment due on Present value. This annuity can be considered as an initial payment of R followed by perpetuity of R of above type. Thus, the present value is given by $P = R + \frac{R}{i}$, where, R = size of each payment i = rate per period

6. SINKING FUND: It is a fund that is accumulated for the purpose of paying off a financial obligation at some future designated date.

The periodic payments of R made at the end of each period required to accumulate a sum of A over n periods with interest charged at the rate i per period is

$A=R\left[\frac{(1+i)^n-1}{i}\right]$ or $A= RS_{\bar{n}/i}$

Where R = Size of each instalment or payment
i = rate per period
n = number of instalments
A = lump sum amount to be accumulated.

7. EMI:- EMI stands for equated monthly instalment. It is a monthly payment that we make towards a loan we opted for at a fixed date of every month.

It can be calculated by two methods

(i)**Flat rate method**: -Let P, I and n are the principal of the loan, the total interest on the principal and number of months in loan period respectively.

EMI is given by the formula $EMI = \left(\frac{P+I}{n}\right)$

(ii) **Reducing balance method**:-

$EMI=R = \frac{P\times i \times(1+i)^n}{(1+i)^n-1}$ or $P\left[\frac{i}{(1-(1+i)^{-n}}\right]$

8. AMORTIZATION PROCESS FORMULAE:-

Principal outstanding at the beginning of k^{th} period= $R\left[\frac{1-(1+i)^{-n+k-1}}{i}\right]$

Total interest paid = nR – P
Where P= amount of the loan
R= size of equal payment
i = rate per period
n = number of equal payments

<u>Very short questions</u>

1. A newspaper printing machine costs ₹4, 80,000 and estimated scrap value of ₹25,000 at the end of its useful life of 10 years. What is its annual deprecation as per linear method?
2. An investment of ₹10,000 becomes ₹60,000 in four years then finds the CAGR.
3. Mr.Satyaveer Singh invests ₹9600 on ₹100 share at ₹80. If the company pays him 18 % dividend, find his rate of return.
4. Mr.Anil has an initial investment of ₹50,000 in an investment plan after 2 years it has grown to ₹60,000. Find his rate of interest.

5. Mrs.suman invests ₹10,000 in 10 percent ₹100 share of a company available at a premium of ₹25. Find her rate of return.
6. A person invested ₹2, 00,000 in a fund for 1 year. At the end of the year the investment was worth ₹ 2, 16,000. Calculate his rate of interest.
7. A machine costing ₹50,000 has a useful life of four years. The estimated scrap value is ₹10,000. Using straight line method, find the annual depreciation.

Short answer questions

1. Mitul invested ₹ 3, 50,000 in a fund. At the end of the year the value of the fund is ₹ 4, 37,500. What is the nominal rate of interest, if the market price is same at the end of the year?
2. Mr. Parteek invested ₹52,000 on ₹100 share at a discount of ₹20 paying 8% dividend. At the end of the year he sales the share at a premium of ₹20. Find his rate of interest.
3. Mrs. Nidhi invests a sum of money in ₹50 shares paying 10%dividend quoted at 20% discount. If her annual dividend is ₹600, calculate her rate of return from the investment.
4. ₹100 shares of a company are sold at a discount of ₹20. If the return on the investment is 15%, find the rate of dividend declared.
5. Mrs. Gupta invested ₹16,500 on ₹100 shares at a premium of ₹10 paying 15% dividend. At the end of the year she sells the shares at a premium of ₹20, find her rate of return.
6. A dining table costing ₹36,000 has a useful life of 15 years. If annual depreciation is ₹2000, find its scrap value using linear method.
7. A vehicle costing ₹9, 00,000 has a scrap value of ₹2, 70,000. If annual depreciation charge is ₹70,000, find its useful life in years.
8. A machine has a scrap value of ₹22,500 after15 years of its purchase. If the annual depreciation charge is ₹8500, find its original cost using linear method.
9. A person invested ₹15000 in a mutual fund and the value of investment at the time of redemption was ₹25000. If CAGR for this investment is 8.88%, calculate the time period for which the amount was invested? [$Given$ log(1.667) = 0.2219 & log(1.089) = 0.037]
10. Find the present value of perpetuity of ₹600 at end of each quarter if money is worth 8% compounded quarterly.
11. Find the present value of an annuity of ₹1000 payable at the end of each year for 5 years if money is worth 6%compounded annually. [$Given$ $(1.06)^{-5} = 0.7473$]
12. What is the face value of a sinking fund that yields a dividend of ₹1800 at 10% semi-annually?
13. ₹ 2, 50,000 cash is equivalent to perpetuity of ₹ 7,500 payable at the end of each quarter. What is the rate of interest convertible quarterly?
14. Mr. Bajrang Lal takes a loan of 2, 00,000 with 10% annual interest rate for 5 years. Calculate EMI under Flat Rate system.
15. What amount is received at the end of every 6 months forever, if Rs 72000 kept in a bank earns 8% per annum compounded half yearly?
16. What sum of money invested now could establish a scholarship of Rs 5000 which is to be awarded at the end of every year forever,if money is worth 8% per annum.
17. Rajani takes a personal loan of Rs 50, 0000 at the rate of 12%per annum for 3 years. Calculate her EMI by using flat rate method.
18. Rahul borrowed Rs 100000 from a co-operative society at the rate of 10% p.a. for 2 years. Calculate the EMI using flat rate method.
19. Sheetal takes a loan of Rs 250000 from a bank at the rate of 14% p.a. for 5 years . Calculate her EMI using flat rate method.

Long answers Questions

1. Mr. Kumar has invested ₹20,000 in year 2014 for 5 years. If CAGR for that investment turned out to be 11.84%. What will be the end balance?
2. Mr. Anuj has bought 200 shares of a company at ₹100 each in year 2000. After selling them he has received ₹30,000 which accounts for 22.47% CAGR. Calculate the number of years for which he was holding the shares.
3. Mr. Rahim invested ₹10,000 in a company's fund. His yearly investment values are shown in the table given below:

Year	0	1	2	3
Amount (In ₹)	10000	13000	11000	9400

   Calculate CAGR of his investment.
4. Surjeet purchased a new house, costing ₹ 40, 00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly instalment is ₹ 30,000, at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? [Use $(1.0075)^{-300} = 0.1062$]
5. 10 years ago, Mr Mehra set up a sinking fund to save for his daughter's higher studies. At the end of 10 years, he has received an amount of ₹ 10, 21,760. What amount did he put in the sinking fund at the end of every 6 months for the tenure, which paid him 5% p.a. compounded semi-annually? [Use $(1.025)^{20} = 1.6386$]
6. A machine costing ₹50,000 is to be replaced at the end of 10 years, when it will have a salvage value of ₹5000. In order to provide money at that time for a machine costing the same amount, a sinking fund is set up. If equal payments are placed in the fund at the end of each quarter and the fund earns 8% compounded quarterly, then what should each payment be? [$Given$ $(1.02)^{40} = 2.208$]
7. A couple wishes to purchase a house for ₹15, 00,000 with a down payment of ₹4, 00,000. If they can amortize the balance at an interest rate 9% per annum compounded monthly for 10 years, find the monthly instalment (EMI). Also find the total interest paid. [$Given$ $(1.0075)^{-120} = 0.4079$]
8. In 10 years, a machine costing ₹ 40,000 will have a salvage value of ₹4,000. A New Machine at that time is expected to sell for ₹52,000. In order to provide funds for the difference between the replacement cost and the salvage cost, a sinking fund is set up into which equal payments are placed at the end of each year. If the fund earns interest at the rate 7% compounded annually, how much should each payment be?
9. A machine cost ₹1, 00,000 and its effective life is estimated to be 12 years. A sinking fund is created for replacing the machine by a new model at the end of its lifetime when its scrap realises a sum of ₹5,000 only. Find what amount should be set aside at the end of each year, out of the profits, for the sinking fund if it accumulates at 5 % effective.
10. A couple wishes to purchase a house for ₹12, 00,000 with a down payment of ₹2, 50,000. If they can amortize the balance at 9% per annum compounded monthly for 20 years
    (i) What is their monthly payment
    (ii) What is the total interest paid? ($a_{\overline{240}/0.0075=111.1449}$)

CASE STUDY

In the year 2000, Ms. Neelam planned to purchase a home. She took a **home loan** of ₹ 3000000 from the State Bank of India at 7.5% p.a. compounded monthly for 20 years.

(https://www.google.com/search?q=HOME&source)

Based on the above information, answer the following questions:

1. Principal outstanding at the beginning of 193rd month was-
   (a) ₹ 385513.82 (b) ₹ 515394.12 (c) ₹ 356432.67 (d) ₹ 410293.41
2. Total interest paid by Ms. Neelam was-
   (a) ₹ 2800276.80 (b) ₹ 3141345.96 (c) ₹ 3200147.83 (d) ₹ 3003415.12
3. The equated monthly instalment paid by Ms. Neelam was-
   (a) ₹ 35288.82 (b) ₹ 37839.85 (c) ₹ 39437.75(d) ₹ 24167.82
4. Interest paid by Ms. Neelam in the 150th payment was-
   (a) ₹ 15712.67 (b) ₹ 9532.76 (c) ₹ 10458.69 (d) ₹ 20132.84
5. Principal paid by Ms. Neelam in 150th payment was-
   (a) ₹ 8455.15 (b) ₹ 14635.06 (c) ₹ 4034.98 (d) ₹ 13709.13

Answer:

Very Short Answers

1. ₹ 45,500	2. $(6^{1/4} - 1) \times 100$	3. 22.5%	4. 20%
5. 8%	6. 8%	7. 10,000	

Short Answers

1. 25%	2. 60%	3. 12.5 %	4. 12%
5. $22\frac{8}{11}$ %	6. ₹ 6000	7. 9 years	8.₹ 1,50,000
9. 5.99 (app. 6 years)	10. ₹ 30,000	11. ₹4211.67	12. ₹36000
13. r=12% p.a.	14. ₹ 5000	15. ₹ 2880	16.₹ 62500
17. EMI= Rs 18888.89	18. ₹ 5000	19. ₹ 7083.33	

Long Answers

1. ₹ 35.000	2. 2 Years	3. -2.05 %	4. ₹424800
5. ₹40000	6. ₹ 745.03	7. ₹ 572020	8 ₹. 3474.12
9. ₹ 5968.8	10. (i) ₹ 8547.20	(ii) ₹ 1101376.17	

CASE STUDY :- (1) d (2) a (3) d (4) a (5) d

LINEAR PROGRAMMING

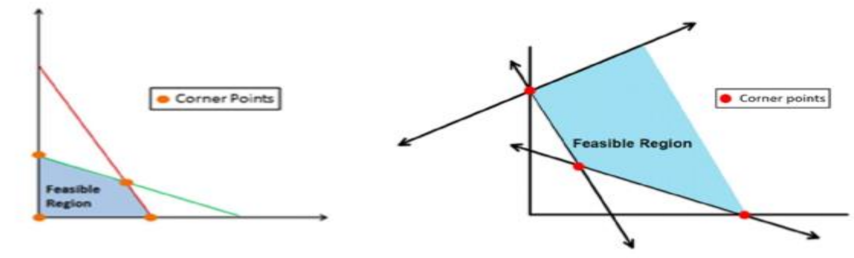
SOME IMPORTANT RESULTS/CONCEPTS

Feasible Region -The common region determined by all the constraints including non-negative constraints $x \geq 0$, $y \geq 0$ of an LPP.

Feasible Solutions - Points within and on the boundary of the feasible region for an LPP represent feasible solutions.

Infeasible Solutions - Any Point outside feasible region is called an infeasible solution.

Optimal Solution - Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.



Bounded (closed) Feasible region   Unbounded (Open) Feasible region

Steps to solve LPP by Corner Point Method:

1. After finding feasible region of the LPP, determine its corner points(vertices).
2. Evaluate the objective function Z=ax+ by at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is bounded, M and m are the maximum and minimum values of Z.

   (ii) In case, the feasible region is unbounded, we have:

   (a) M is the maximum value of Z, if the open half plane determined by ax+ by >M has no point in common with the feasible region. Otherwise, Z has no maximum value.

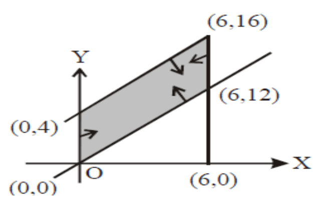
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by ax+ by<m has no point in common with the feasible region. Otherwise, Z has no minimum value.

Steps to solve ISO-Profit/ ISO-Cost Method:

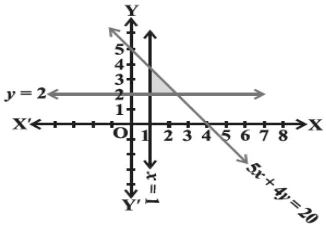
1. As discussed in Corner-Point method, identify the feasible region and extreme (corner) points.
2. Give some convenient values to Z and draw the line so obtained in xy- plane.
3. If the objective function is to be maximized, then draw lines parallel to the line in step 2.
4. Obtain a line which is farthest from the origin and has at least one point common to the feasible region.
5. If the objective function is to be minimized, then draw lines parallel to the line in step 2 and obtain a line which is nearest to the origin and has at least one point common to the feasible region.
6. Find the co-ordinates of the common point obtained in step 4(in case of maximum) or step 5(in case of minimum). The point so obtained determines the optimal solution and the value of the objective function at these points give the optimal solution.

Short answer questions

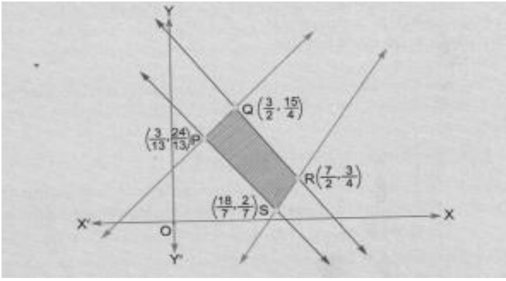
1. The feasible region for LPP is shown shaded in the figure. Let $Z = 3x - 4y$ be the objective function, then write the maximum value of Z .



2**.** Write the linear inequations for which the shaded area in the following figure is the solution set.



3. In figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of Z=x+2y.



4. One kind of cake requires 300 g of flour and 15g of fat, another kind of cake requires 150g of flour and 30g of fat. Find the maximum number of cakes which can be made from 7.5kg of flour and 600g of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an L.P.P.
5. A book publisher sells a hard cover edition of a book for ₹ 72 and a paperback edition for ₹ 40. In addition to a fixed weekly cost of ₹ 9,600, the cost of printing hardcover and paperback editions are ₹ 56 and ₹ 28 per book respectively. Each edition requires 5 minutes on the printing machine whereas hardcover binding takes 10 minutes and paperback takes 2 minutes on the binding machine. The printing machine and the binding machine are available for 80 hours each week. Formulate the linear programming problem to maximise the publisher's profit.

Answers

1. 0 2. $5x+4y \leq 20$, $x \geq 1$, $y \geq 2$ 3. Max. Z=9 at point Q, Min. Z=22/7 at point S

4. Max $Z=x+y$ subject to $2x + y \leq 50$, $x + 2y \leq 40$, $x, y \geq 0$

5. Maximum $Z=16x + 12y - 9600$ Subject to: $x + y \leq 960$, $5x + y \leq 2400$, $x, y \geq 0$

Long Answer Questions

1. Solve the following Linear Programming Problems graphically:
   Maximise $Z = 5x + 3y$ Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

2. Maximize $Z = x + y$ Subject to constraints: $x, y \geq 0$, $x - y \leq -1$, $x \geq y$

3. Minimize $Z = 3x + 5y$ Subject to constraints: $x, y \geq 0$, $x + 3y - 3 \geq 0$, $x + y - 2 \geq 0$.
4. (Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamins A. Each packet of the same quantity of food Q contain 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of

calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food are used to minimize the amount of Vitamin A in the diet? What is the minimum amount of Vitamin A?

5. Anil wants to invest at most Rs12,000 in bonds A and B. According to the rules, he has to invest at least Rs 2,000 in Bond A and at least Rs4,000 in Bond B. If the rate of interest on Bond A is 8% per annum and on Bond B is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.
6. A cylinder manufacturer makes small and large cylinders from a large piece of cardboard. The large cylinder requires 4 sq. m and the small cylinder requires 3 sq. m of cardboard. The manufacturer is required to make at least 3 large cylinders and at least twice as many small cylinders as large cylinders. If 60 sq. m of cardboard is in stock, and if the profits on small and large cylinders are Rs. 20 and Rs. 30 per cylinders respectively, how many of each type should be made in order to maximize the total profit?
7. An aeroplane can carry a maximum of 200 passengers. A profit of Rs400 is made on each first class ticket and a profit Rs300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by second class then by first class. Determine how many tickets f each types must be sold to maximize profit for the airline. Form an LPP and solve it graphically.
8. There is a factory located at each of the two places P and Q .From these locations, a certain commodity is derived to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5 ,5 and 4 units of the commodity while the production capacity of the factories at P and Q are 8 and 6 units respectively. The cost of transportation per unit is given below:

From/to	Costs (in Rs)		
	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? Formulate above as a linear programming problem.

9. Solve the following Linear Programming Problem ISO-Profit method. Maximize $Z = 15x + 10y$ Subject to $4x + 6y \leq 360$, $3x \leq 180$, $5y \leq 200$, $x \geq 0$ and $y \geq 0$.
10. Solve the following Linear Programming Problem using ISO-Cost method. Minimize $Z = 18x + 10y$ Subject to $4x + y \geq 20$, $2x + 3y \geq 30$, $x \geq 0$ and $y \geq 0$.

Answers

1. Maximum Z=235/19 at (20/19, 45/19) 2.No possible feasible region hence Z cannot be maximized
3. Minimum value of Z is 7 when x = 3/2, y = 1/2. 4. 150 units (15, 20) 5. (2000, 10000) maximum profit is ₹1,160
6. Small Cylinders – 12, Large cylinders – 6, Maximum Profit = Rs. 420
7. ₹ 64,000 at (40,160)
8. Minimize $Z = x - 7y + 190$ Subject to the constraints: $x \geq 0$, $y \geq 0$ $x + y \leq 8$, $x + y \geq 4$, $x, y \leq 5$.
9. Maximum Profit obtained is $Z = 1100$ at $x = 60$ and $y = 20$.
10. The minimum value of Z is 134 at $x = 3$ and $y = 8$

CASE STUDY

A train can carry a maximum of 300 passengers. A profit of Rs. 800 is made on each executive class and Rs. 200 is made on each economy class. The IRCTC reserves at least 40 tickets for executive class.

However, at least 3 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class ticket is Rs. x and that of economy class ticket is Rs. y. Optimize the given problem.

Based on the above information, answer the following questions.

1 The objective function of the LPP is:

(a) Maximise Z = 800x + 200y (b) Maximise Z = 200x + 800y

(c) Minimise Z = 800x + 200y (d) Minimise Z = 200x + 800y

2 Which among these is a constraint for this LPP?

(a) x+y = 300 (b) $y \geq 3x$

(c) $x \leq 40$ (d) $y \leq 3x$

3 Which among these is not a corner point for this LPP?

(a) (40,120) (b) (40, 260)

(c) (30, 90) (d) (75, 225)

4 The maximum profit is:

(a) Rs.56000 (b) Rs. 84000

(c) Rs. 205000 (d) Rs. 105000

5 Which corner point the objective function has minimum value?

(a) (40,120) (b) (40, 260)

(c) (30, 90) (d) (75, 225)

ANSWERS

1 (a) 2 (b) 3 (c) 4 (d) 5 (a)